The Continuum – on one sheet of paper

A point marks a boundary between two extended parts of the continuum. A point has no magnitude. When a boundary point is removed (in thought alone) from the continuum all the extension is left behind. The continuum is thereby not diminished one iota by this operation.

Infinity does not make any difference. When an infinite set of boundary points is removed from the line the entire extension remains. Therefore, to construct the arithmetical continuum there must be two primitive notions: boundary points and extensions. We need:

Axiom of Indestructibility of Extension: Every proper part of an extended portion of space is extended.
Or in the words of Kant: "**Space … consists solely of spaces, time solely of times**. Points and instants are only limits, that is, mere positions which limit space and time."

Why you are receiving this message
The attempt to understand analysis using set theory alone is a 150 year error of mathematical analysis, and the truth must come to light. This error is causing havoc in mathematics, theoretical physics, and philosophy.
(Because of meddlers, it is also necessary to establish immediately one's authorship.)

Consequences
(1) Since set theory permits of only one primitive of set membership, it follows that set theory is inadequate to describe the continuum. The view that set theory is the foundation of mathematics is false. (It is an illusion anyway to claim that set theory has only one primitive: the concept of the actual infinity is not analytically contained in the concept of membership.)
(2) A digital computer cannot be constructed upon a system of more than one primitive. Therefore, the claims of strong AI are false: the human mind is not a digital computer.

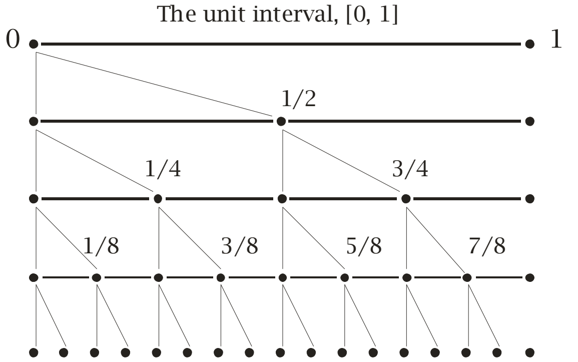
Technical
(1) The Cantor set is perfect (equal to its own closure), non-continuous, nowhere dense (totally disconnected) and has outer content (measure) equal to zero. Since the unit interval $[0,1]$ is perfect, continuous, has a dense subset and has unit measure, the Cantor set is not homeomorphic to it. Thus, the Cantor set is not the arithmetic continuum. It is a representing set for the continuum, in the sense that it has as many elements as there are points on the real line corresponding to real numbers.
(2) When extensions are shrunk in the limit to incommensurable objects, they become "extension points". The taking of limits partitions the continuum into two infinite collections of limits – one of boundary points of absolutely zero measure and another of incommensurable extension points (continua).
(3) The arithmetical continuum is not given in intuition, and is a construct of science. Hence the structure of the continuum is an empirical question and there may be more than one structure proposed. The arithmetical continuum is the structure derived by exponentiation of the one-point compactification of its skeleton, which is a dense subset of a separable space, under a countable reordering as in Cantor diagonalisation.

Remark
Set theory is based on the actual infinite, number theory on the potential infinite; therefore, number theory cannot be embedded in set theory.

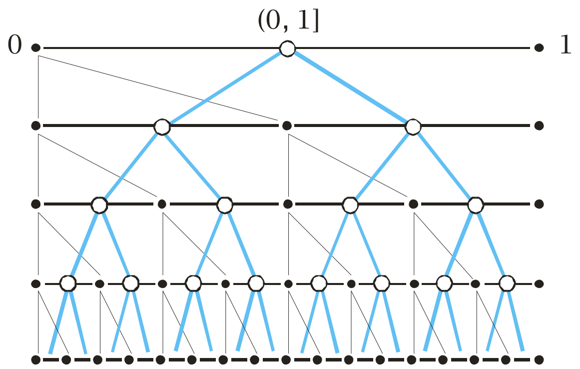
MELAMPUS, Februrary 2019

See reverse for heuristic diagrams.

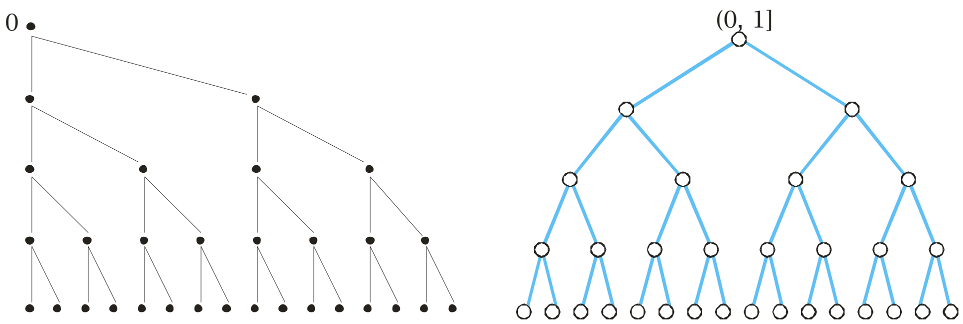
Heuristic diagrams



Extracting boundary points systematically from the unit interval.



Black dots are boundary points; open circles are the extensions left behind when the boundaries are removed.



The arithmetical continuum is a union of two trees, both in the limit are Cantor sets, one of boundaries, the other of extensions.

$\mathbf{2}^{\omega} \oplus \mathbf{2}^{\omega}$

Representation by a single Cantor set is not identical to arithmetical continuum, as extensions cannot be made into boundaries, and vice-versa. Technically, two trees may be combined into one, as one representation – conceptually, the extensions remain distinct from the boundaries.