Solution to the Five State Hold outs

MELAMPUS

Abstract

The Halting Problem for each of the five-state hold out Turing machines as specified in Kellett [2005] is solved using the method of exits described in the *Solution to the Halting Problem* [Melampus, 2019].

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Contents

References

Kellett, Owen [2005]

A multi-faceted attack on the Busy Beaver Problem. Rensselaer Polytechnic Institute, Troy, New York. 2005.

Melampus [2019]

Solution to the Halting Problem. Black's Academy Limited, www.melampus.name, 2019

Preface

What is a five state hold out?

A five–state hold out is a Turing machine of five states that Kellett [2005] was unable to classify for certain as either a halting or non-halting machine for standard starting configuration.

It is assumed that the reader is familiar with Turing machines and the definition of the Halting Problem. This paper is a supplement to the paper *Solution to the Halting Problem* [Melampus 2019]. It illustrates that paper by providing in the first section fully annotated solutions to selected problems, and in the second section notes to the other solutions. In the second section, the reader must refer to Kellett [2005] for a description of each Turing machine under discussion.

For the reader familiar with Turing machines this paper can be read independently of the *Solution to the Halting Problem*. A description of a Turing machine is provided in the appendix to the *Solution to the Halting Problem*.

The objective is to illustrate how in principle the Halting Problem for any Turing Machine of any size is solvable by the methods introduced in the *Solution to the Halting Problem.* The reader is invited to read the *Solution to the Halting Problem* for a discussion of the apparent contradiction between this result and the "impossibility proof" for the Halting Problem. That paper also discusses the philosophical consequences of the result.

The method of exits

The solutions employ the method of exits described in *Solution to the Halting Problem* [Melampus, 2019]

1. By backward trace through the machine from its exits, either directly or by iteration of the idea of adding one more state to a pre-existing machine, construct a tree. The tree is shown to be always finite in the main text – that is, it has branches that terminate, hence has finite depth and never branches infinitely, hence has finite width.

2. From the tree construct the complete criterion for the machine. This may also be constructed in stages through iteration.

3. The complete criterion determines all the halting configurations of the machine. The standard starting configuration is $S_0 = \overline{\overline{0}}_0$. Hence, the machine halts if, and only if, the standard starting configuration appears among the halting configurations of the complete criterion. If it does not halt for standard starting configuration, then for that configuration it has entered the infinitely recurring cycle of the machine.

Finite does not mean the same as "small"

Although the method of exits generates a tree of finite width and depth, owing to the possibly large number of permutations of input configurations, the tree may be very large. Hence, it is not always possible to generate the whole tree when working "by hand". The process would appear to be an algorithm, and be better conducted by a computer. Herein the solutions are "by hand", so on most occasions the complete criterion for the five-state machine is not given, as being prone to too much human error. Sometimes contradiction arguments and heuristic methods are used to quickly determine whether the machine does or does not halt.

Caveat emptor

Any solution by hand is subject to human error. The method of exits involves reversing symbols – reading a "move left" as a "move right" and vice-versa; marking a scanned state and a scanned symbol, and so forth. These reversals are mentally tiring and prone to error. No fundamental principle is involved should a casual error arise. Hence, subject to this caveat, the Halting Problem for all five-state machines is solved, and the productivity of five-state machines is 11, which is the productivity of the B5 Champion.

Fully annotated solutions

Hold out no. 4. Kellett B.1

This machine has one exit at X3 (*figure 4.1*). Direct solution could be "tricky" owing to the three inputs at state Q4. We solve by considering the sub-machine, T3, obtained by removing states Q3 and Q4.

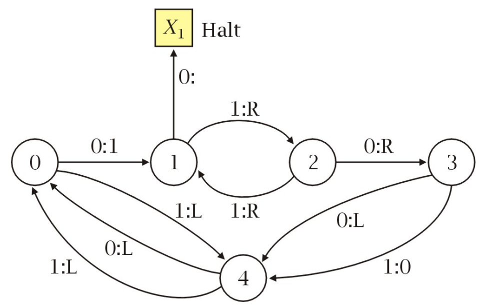


Figure 4.1. Hold out no. 4.

T3 (*figure 4.2*) has three exits at X0, X1 and X2. We solve by the method of exits to write the complete criterion. In the solution, an impossible configuration is shown in red. These denote termini to the branches of the tree generated by the method of exits. Certain configurations cannot be traced further backwards; these are marked in red by END.

Exit at X_0	Exit at X_1				Exit at X_2		
1_0	0_1				0_2		
	$0:1$	1_20			1_10		
		1_110			0_00	1_210	
		0_010	1_2110		END	1_1110	
		END	1_11110			0_0110	1_21110
			0_01110	1_211110		END	1_111110
			$0_0\overline{1}110$				$0_0\overline{11}0$
			$1_1\overline{1}110$				$1_1\overline{11}0$
			$1_2\overline{11}0$				$1_2\overline{1}110$

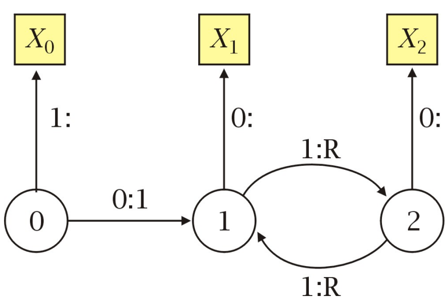


Figure 4.2. T3 for no. 4.

Complete criterion for T3

S_0		S_1		S_2	
1_0	$H = \boxed{1_0}$	0_1	$H = \boxed{0_1}$	1_20	$H = 1\boxed{0_1}$
0_010	$H = 11\boxed{0_1}$	1_110	$H = 11\boxed{0_1}$	$1_2\overline{11}0$	$H = 1\ \overline{11}\ \boxed{0_1}$
$0_0\overline{1}110$	$H = 1\overline{11}1\boxed{0_1}$	$1_1\overline{1}110$	$H = 1\overline{11}1\boxed{0_1}$	0_2	$H = \boxed{0_2}$
0_00	$H = 1\boxed{0_2}$	1_10	$H = 1\boxed{0_2}$	1_210	$H = 11\boxed{0_2}$
$0_0\overline{11}0$	$H = 1\ \overline{11}\ \boxed{0_2}$	$1_1\overline{11}0$	$H = 1\ \overline{11}\ \boxed{0_2}$	$1_2\overline{1}110$	$H = 1\overline{11}1\boxed{0_2}$

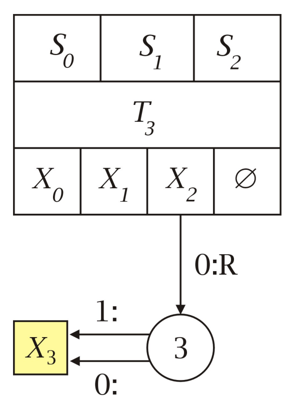


Figure 4.3 T4 for no. 4.

We obtain T4 (*figure 4.3*) by adding the state Q3 to T3. The complete criterion is automatic. T4 can only exit at X3 if T3 exits at X2 on a 0; so in the complete criterion for T3 we replace every exit at X2 by an exit at X3, and the configuration at X3 depends on the initial tape configuration.

Hold out no. 4. Kellett B.1 *continued*

Complete criterion for T4

S_0		S_1		S_2		S_3
1_0	$H = \boxed{1_0}$	0_1	$H = \boxed{0_1}$	1_20	$H = 1\boxed{0_1}$	0_3
0_010	$H = 11\boxed{0_1}$	1_110	$H = 11\boxed{0_1}$	$1_2\overline{11}0$	$H = 1\,\overline{11}\,\boxed{0_1}$	1_3
$0_0\overline{11}10$	$H = 1\overline{11}1\boxed{0_1}$	$1_1\overline{11}10$	$H = 1\overline{11}1\boxed{0_1}$	0_20	$H = 0\boxed{0_3}$	
0_000	$H = 10\boxed{0_3}$	1_100	$H = 10\boxed{0_3}$	1_2100	$H = 110\boxed{0_3}$	
0_001	$H = 10\boxed{1_3}$	1_101	$H = 10\boxed{1_3}$	1_2101	$H = 110\boxed{1_3}$	
$0_0\overline{11}00$	$H = 1\,\overline{11}0\boxed{0_3}$	$1_1\overline{11}00$	$H = 1\,\overline{11}0\boxed{0_3}$	$1_2\overline{11}100$	$H = 1\overline{11}10\boxed{0_3}$	
$0_0\overline{11}01$	$H = 1\,\overline{11}0\boxed{1_3}$	$1_1\overline{11}00$	$H = 1\,\overline{11}0\boxed{1_3}$	$1_2\overline{11}100$	$H = 1\overline{11}10\boxed{1_3}$	

T5 is obtained from T4 by addition of state Q4 (*figure 4.4*). In T5 there is only one exit at X1. Supposing the machine exits at X1 scanning a 0 and halts, then the inputs at S1 and S2 that lead to this exit in T4 may be ignored, because those inputs are not in standard configuration. Only at S0 can we have standard configuration, or a loop backwards to state Q4 that may eventually lead to the standard configuration at S0. We use the method of exits to draw the tree, tracing backwards.

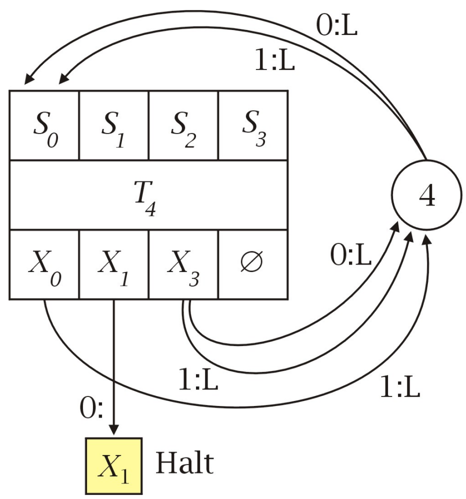


Figure 4.4. T5 for no. 4.

Exit at X_1

0_1				
S_1	$X_1 = 110_1$		$X_1 = 1\overline{11}10_1$	
S_2	$S_0 = 0_010$		$S_0 = 0_01\overline{11}0$	
S_3	01_40		$01_4\overline{11}0$	
	010_3	$1:L$	$01\overline{1_01}0$	$01\overline{1_3}10$
	S_1		$01\overline{11_4}0$	END
	S_2		$01\overline{11}0_3$	$01\overline{11}\,\overline{1_3}10$
	S_3		S_1	END
	0_000		S_2	
	$0_0\overline{11}00$		S_3	
			0_000	
			$0_0\overline{11}00$	

In the solution, the irrelevant inputs and the impossible configurations are shown in red. These denote termini to the branches of the tree generated by the method of exits. Certain configurations cannot be traced further backwards; these are marked in red by END. If T5 halts for any input not at S3 then it does so scanning a 0 with a 1 to the left. But it is impossible to reach such a configuration from S0. Hence, the input of standard starting configuration is shown explicitly by this method to be impossible. Therefore, T5 = Hold out no. 4 does not halt for standard starting configuration.

Standard starting configuration: $S_0 = \overline{\overline{0}}_0$.

Hold out no. 15. Kellett B2.

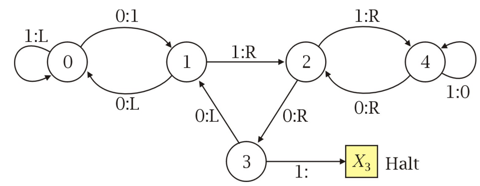


Figure 15.1. Hold out no. 15

This machine has one exit at X3 (*figure 15.1*). We solve by considering the sub-machine, T4, obtained by removing state Q3.

T4 has one exit at X2 (*figure 15.2*). We solve by the method of exits to write the complete criterion. In the solution, an impossible configuration is shown in red. These configurations denote termini to the branches of the tree generated by the method of exits.

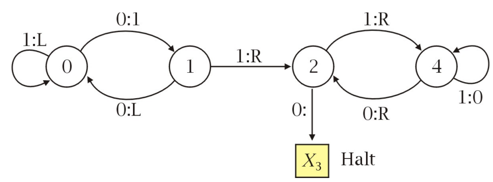


Figure 15.2. T4 for no. 15.

Exit at X_2

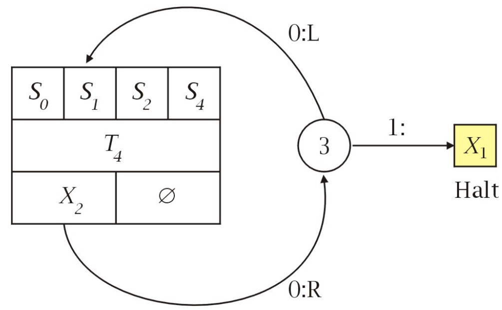
0_2											
$1_1 0$			$0_4 0$								
$0_0 0$			$1_4 0$		$1_2 00$						
$1_0 0$		00_1	$1_2 10$		$1_1 100$		$0_4 100$				
10_1	$1:L$	$0:1$	$1_1 110$	$0:R$	$0_0 100$		$1_2 0100$			$1_4 100$	
$0:1$			$0_1 110$		$01_0 00$	$0:L$	$1_1 10100$	$10_4 100$		$1_2 1100$	$1:0$
			$01_0 10$	$0:L$	$010_1 0$		$0_0 10100$	$11_4 100$		$1_1 11100$	$0:R$
			$011_0 0$	$0:L$	$0:1$		$01_0 0100$	$1_2 1100$	$1:0$	$0_0 11100$	
			0110_1	$1:L$			$010_1 100$	$1_1 11100$		$01_0 1100$	$0:L$
			$0:1$				$0:1$	$0_0 11100$		$011_0 100$	$0:L$
								$01_0 1100$	$0:L$	$0111_0 00$	$0:L$
								$011_0 100$	$0:L$	$01110_1 0$	$1:L$
								$0111_0 00$	$0:L$	$0:1$	
								$01110_1 0$	$1:L$		
								$0:1$			

Complete criterion for T4

S_0		S_1		S_2		S_4	
0_2	$H = \boxed{0_2}$	10_1	$H = 1\boxed{0_2}$	$1_2 10$	$H = 10\boxed{0_2}$	$0_4 0$	$H = 0\boxed{0_2}$
$0_0 0$	$H = 1\boxed{0_2}$	00_1	$H = 1\boxed{0_2}$	$1_2 00$	$H = 10\boxed{0_2}$	$1_4 0$	$H = 0\boxed{0_2}$
$1_0 0$	$H = 1\boxed{0_2}$	$1_1 110$	$H = 110\boxed{0_2}$	$1_2 0100$	$H = 1010\boxed{0_2}$	$0_4 100$	$H = 010\boxed{0_2}$
$01_0 10$	$H = 110\boxed{0_2}$	$0_1 110$	$H = 110\boxed{0_2}$	$1_2 1100$	$H = 1010\boxed{0_2}$	$10_4 100$	$H = 1010\boxed{0_2}$
$11_0 10$	$H = 110\boxed{0_2}$	0110_1	$H = 110\boxed{0_2}$			$11_4 100$	$H = 1010\boxed{0_2}$
$111_0 0$	$H = 110\boxed{0_2}$	$1_1 110$	$H = 110\boxed{0_2}$			$1_4 100$	$H = 010\boxed{0_2}$
$0_0 100$	$H = 110\boxed{0_2}$	$010_1 0$	$H = 110\boxed{0_2}$				
$0_0 100$	$H = 110\boxed{0_2}$	$1_1 10100$	$H = 11010\boxed{0_2}$				
$01_0 00$	$H = 110\boxed{0_2}$	$110_1 100$	$H = 11010\boxed{0_2}$				
$0_0 10100$	$H = 11010\boxed{0_2}$	$1_1 11100$	$H = 11010\boxed{0_2}$				
$01_0 0100$	$H = 11010\boxed{0_2}$	$01110_1 0$	$H = 11010\boxed{0_2}$				
$0_0 11100$	$H = 11010\boxed{0_2}$						
$01_0 1100$	$H = 11010\boxed{0_2}$						

Hold out no. 15 *continued*.

We observe from the complete criterion for T4 that it only ever halts if scanning a 0 with a configuration determined to the left of that 0. But if T5 (*figure 15.3*) halts it must scan a 1 to the right of that 0. Hence, whether T5 halts or not is determined solely by the presence of that 1 in the initial tape configuration. Standard starting configuration does not have it; therefore T5 = No. 15 does not halt for standard starting configuration.

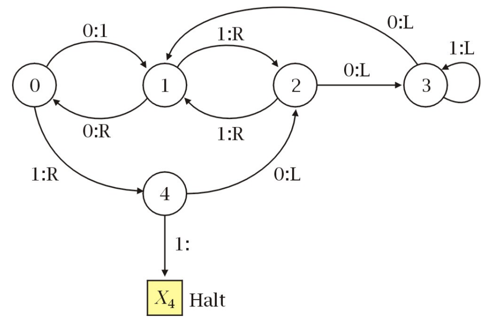


Fjgure 15.3. T5 for hold out no. 15

Standard starting configuration: $S_0 = \overline{\overline{0_0}}$.

Hold out no. 19. Kellett B3.

The solution to no. 20 is given in the text *Solution to the Halting Problem*.



Fjgure 19.1. Hold out no. 19

Hold out no. 19 contains the same sub-machines analysed in the solution to no. 20, designated T3 and T4 in that solution. No. 19 differs from no. 20 only in having the instruction 0:R from Q4 to Q1 in no. 20 replaced by the instruction 0:L from Q4 to Q2. The print out of the action of this machine upon the standard configuration is identical up to line 39, when both machines enter state Q4 for the first time; thereafter, they differ.

The analysis of the halting behaviour of both machines is identical. No 19 does not halt because in tracing back by the method of exits from X4 we obtain a tape contradiction, subject to the hypothesis that the computer started in standard configuration. The same observation about the inability of the machine to erase a 1 once it has been written to the tape also applies. Please see the solution to no. 20.

This machine has the same complete criterion for the T3 machine defined in the solution to no. 20. It also has the same trace for the method of exits for the T4 machine, which confirms the impossibility of the machine reaching a halting configuration on the assumption that it started in standard starting configuration.

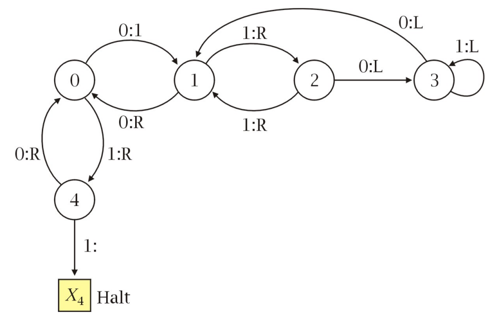
Standard starting configuration: $S_0 = \overline{\overline{0}}_0$.

No. 19 does not halt for standard starting configuration.

Hold out no. 21. Kellett B5.

The solution to no. 20 is given in the text *Solution to the Halting Problem*.

This machine contains the same sub-machines analysed in the solution to no. 20, designated T3 and T4 in that solution. No. 21 differs from no. 20 only in having the instruction 0:R from Q4 to Q1 in no. 20 replaced by the instruction 0:R from Q4 to Q0. The print out of the action of this machine upon the standard configuration is identical up to line 39, when both machines enter state Q4 for the first time; thereafter, they differ.



Fjgure 21.1. Hold out no. 21

The analysis of the halting behaviour of both machines is identical. However, we can make a sharp observation that illustrates the whole point about the inductive method for proving the Halting Problem.

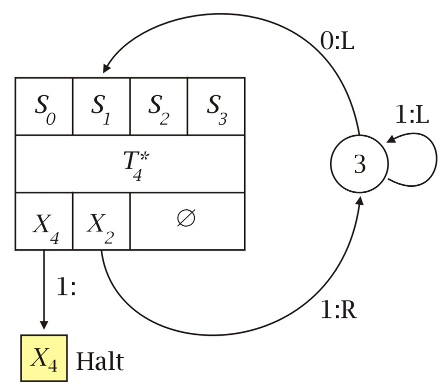


Figure 21.2. T4* for no. 21

Using the same concept of a submachine T4* as in the solution to No. 20 (*figure 21.2*) suppose that no. 21 halts for standard configuration. Since the input of standard configuration at S0 does not lead directly to the exit at X4, this is only possible if starting at S0 in standard configuration, there is a path from S0 to X2, thence to Q3 back to S1. Then S1 may feed forwards through T4* to X2 and through Q3 to S1 again. Such a loop may be repeated finitely many times before eventually the input at S1 leads through T4* to X4. Now we apply the method of exits, supposing no. 21 halts at X4.

Exit and halt at X_4

1_4

$1_0 1$

$0_1 11$

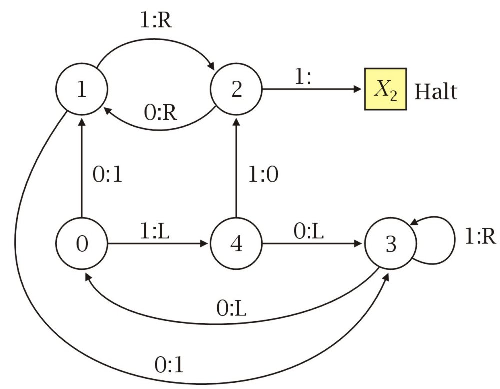
$0:L$

The assumption that we have entered S1 from state Q3 already generates a tape contradiction. Therefore, this machine can never reach that state. Machine no. 21 does not halt for standard starting configuration.

Standard starting configuration: $S_0 = \overline{\overline{0_0}}$.

Hold out no. 49. Kellett B6.

It is relatively straightforward to prove that this machine does not halt by directly constructing its complete criterion by the method of exits.



Fjgure 49.1. Hold out no. 49

Exit at X_2

1_2		
$1_1 1$	$1:0$	
$0_0 1$	$0_2 11$	
$0:L$	$1_1 011$	$1_4 11$
	$0_0 011$	$11_0 1$
	$00_3 11$	$0:L$
	$1:R$	
	$0:L$	
	$0:1$	

Unlike the case for so many of these "difficult" problems, here the tree reaches tape contradictions relatively quickly. Since we have no feedback loop to consider in this case, as we are writing its complete criterion directly, we see automatically that the standard configuration is not among the list of halting configurations. Therefore, no. 49 does not halt for standard starting configuration.

Standard starting configuration: $S_0 = \overline{\overline{0_0}}$.

Complete criterion

S_0		S_1		S_2		S_3	
$0_0 1$	$H = 1\boxed{1_2}$	$1_1 1$	$H = 1\boxed{1_2}$	1_2	$H = \boxed{1_2}$	$00_3 11$	$H = 101\boxed{1_2}$
$0_0 011$	$H = 101\boxed{1_2}$	$1_1 011$	$H = 101\boxed{1_2}$	$0_2 11$	$H = 01\boxed{1_2}$	S_4	
$11_0 1$	$H = 01\boxed{1_2}$					$1_4 11$	$H = 01\boxed{1_2}$

If the reader cares to examine the print out of the action of no. 49 on the Turing tape following the input of standard configuration, he or she will see that none of these halting configurations appear in it.

Hold out no. 55. Kellett A.80

No. 55 (*figure 55.1*) has only one exit. We approach this by solving for a 4-state sub-machine first, obtained by removing state Q4. We denote this machine by T4.

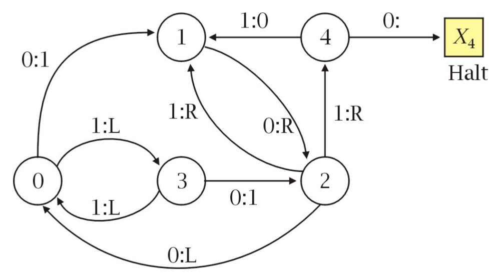


Figure 55.1. Hold out no. 55.

T4 (*figure 55.2*) halts in state Q2 on a 1. We solve by the method of exits to write the complete criterion. In the solution below, impossible configurations are shown in red. These denote termini to the branches generated by the method of exits. Loop configurations are shown in blue.

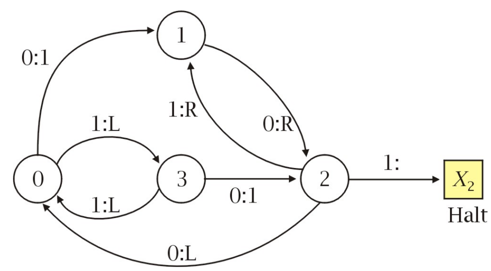


Figure 55.2. T4 for no. 55.

Exit at X_2

1_2					
0_3				$0_1 1$	
01_0				$1_2 01$	$0:1$
011_3		010_2		$0_1 101$	$0_3 01$
0111_0		$0:1$	$0:R$	$1_2 0101$	$1:L$
01111_3	01110_2			$0_1 10101$	
011111_0	$0:1$			$1_2 \overline{01}$	
0111111_3	0111110_2			$0_1 \overline{10}1$	
$0\overline{1}11_0$	$0:1$				
$01\overline{1}11_3$	$0\overline{1}110_2$				

Complete criterion for T4

S_0

$01_0 \quad H = \boxed{1_2}1$

$0\overline{1}11_0 \quad H = \boxed{1_2}\overline{1}11$

S_1

$0_1 1 \quad H = 0\boxed{1_2}$

$0_1\overline{10}1 \quad H = 0\overline{10}\boxed{1_2}$

S_2

$1_2 \quad H = \boxed{1_2}$

$1_2\overline{10}1 \quad H = \overline{10}\boxed{1_2}$

$0\overline{1}110_2 \quad H = \boxed{1_2}\overline{1}110$

S_3

$0_3 \quad H = \boxed{1_2}$

$01\overline{1}11_3 \quad H = \boxed{1_2}\overline{1}110$

$0_3 01 \quad H = 10\boxed{1_2}$

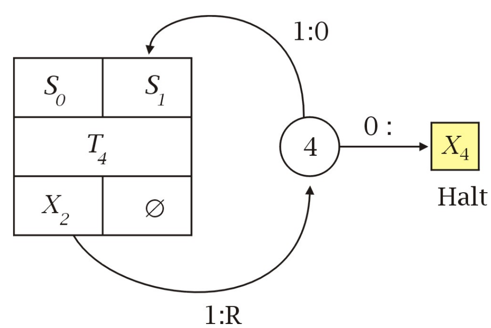


Figure 55.3. T5 for no. 55.

Since T4 does not halt for standard starting configuration, T5 (*figure 55.3*) does not halt for standard starting configuration.

Standard starting configuration: $S_0 = \overline{\overline{0}}_0$.

Hold out no. 56. Kellett A.81

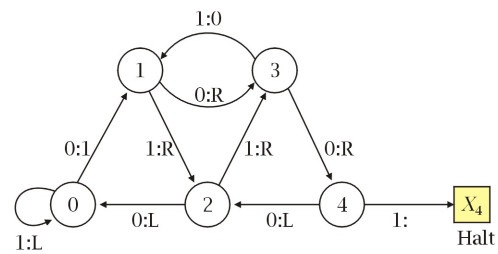


Figure 56.1. Hold out no. 56

No. 56 (*figure 56.1*) has only one exit. We approach this by solving for a 3-state sub-machine first, obtained by removing states Q0 and Q4. We denote this machine by T3.

T3 (*figure 56.2*) has two exits at X2 and X3. We solve by the method of exits to write the complete criterion. In the solution below, an impossible configuration is shown in red. These denote termini to the branches of the tree generated by the method of exits. Loop configurations are shown in blue, which also denote termini of the tree.

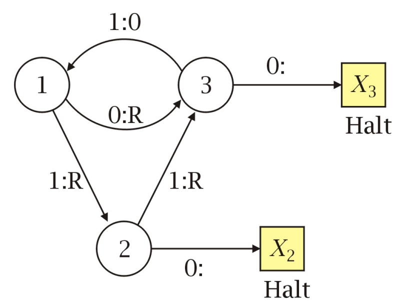


Figure 56.2. T3 for no. 56

Exit at X_3	Exit at X_2
0_3	0_2
$0_1 0$	$1_1 0$
$1_3 0$	$1:0$
$0_1 10$	
$1_3 10$	
$1_3 \bar{1} 10$	
$0_1 \bar{1} 0$	
$\bar{\bar{1}} 1_3 \bar{1} 0$	

Complete criterion for T3

S_1		S_2		S_3	
$0_1 0$	$H = 0\boxed{0_3}$ (1)	0_2	$H = \boxed{0_2}$	$1_3 0$	$H = 1\boxed{0_3}$ (3)
$0_1 \bar{1} 0$	$H = 0\bar{1}\boxed{0_3}$ (2)			$1_3 \bar{1} 10$	$H = \bar{1}\boxed{0_3}$ (3)
$1_1 0$	$H = 1\boxed{0_2}$			$\bar{\bar{1}} 1_3 \bar{1} 0$	$H = \bar{\bar{1}}\boxed{0_3}$ (3)

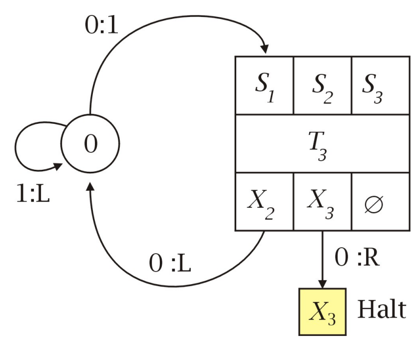


Figure 55.3. T4 for no. 56.

We obtain T4 by adding the state Q0 to T3. To obtain its complete criterion we again use the method of exits. From the complete criterion for T3 there are three possible ways in which T4 may exit at X3. These are shown in green.

Standard starting configuration: $S_0 = \overline{\overline{0_0}}$.

Exit at X_3

(1)	(2)	(3)		
00_3	$0\bar{1}0_3$	10_3		
$0_1 0$	$0_1 \bar{1} 0$	$1_3 0$	$1_3 \bar{1} 10$	$\bar{\bar{1}} 1_3 \bar{1} 0$
$0:1$	$0:1$			

Impossible inputs are shown in red. These indicate that there is no path from the standard configuration at S0 to the input S1. This applies to paths (1) and (2) in green. For the third path from an input to the exit at X3, path (3), none of these are inputs at S1, and hence not possible paths from S0. Hence T5 does not halt.

Hold out no. 57. Kellett A.82

No. 57 (*figure 57.1*) has one exit at X3. Solution directly is tricky, because it contains loops of every possible length—1,2,3,4 and 5-cycles; hence, there are many permutations. We solve in two stages, and firstly, by considering the sub-machine, T3, obtained by removing states Q0 and Q4.

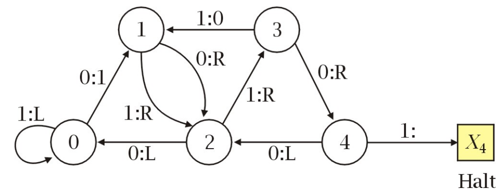


Figure 57.1. Hold out no. 57.

T3 (*figure 57.2*) has two exits at X2 and X3. We solve by the method of exits to write the complete criterion. There is no input at state Q1, hence once the backward trace reaches that state, the branch of the tree ends; this is shown in red in the table below.

Exit at X_2		Exit at X_3	
0_2		0_3	
$1_1 0$	$0_1 0$	$1_2 0$	
END	END	$0_1 10$	$1_1 10$
		END	END

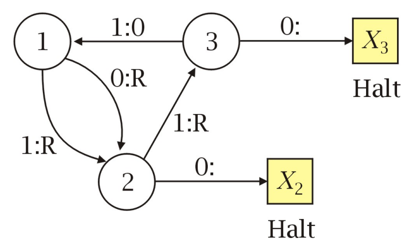


Figure 57.2. T3 for no. 57.

Complete criterion for T3

S_1		S_2		S_3	
$0_1 10$	$H = 01\boxed{0_3}$	$1_3 0$	$H = 1\boxed{0_3}$	0_3	$H = \boxed{0_3}$
$1_1 10$	$H = 11\boxed{0_3}$	0_2	$H = \boxed{0_2}$		
$1_1 0$	$H = 1\boxed{0_2}$				
$0_1 0$	$H = 0\boxed{0_2}$				

We obtain T4 (*figure 57.3*) by adding the state Q0 to T3, and find the complete criterion by the method of exits. In the solution, an impossible configuration is shown in red. These denote termini to the branches of the tree generated by the method of exits. The backward trace shows that an input at S0 in standard configuration leads to T4 halting at X2. Therefore, at this stage of the analysis it is possible that T5 halts for standard configuration.

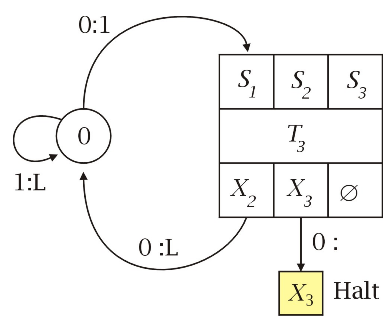


Figure 57.3. T4 for no. 57.

Exit at X_3			
010_3	110_3		10_3
$0_1 10$	$1_1 10$		$1_2 0$
$0:1$	$0_1 10$		$\varnothing$
	$01_0 0$	$0:L$	
	010_2		
	$01_1 0$		
	$00_0 0 \rightarrow SC = \overline{\overline{0}}_0$		
	$1:L$	000_2	

$$SC = \overline{\overline{0}}_0 \rightarrow X_3 = 11\boxed{0_3}$$

Hold out no. 57. Kellett A.82. *Continued.*

Complete criterion for T4

S_0

$0_0 10 \quad H = 11\boxed{0_3}$

$01_0 0 \quad H = 11\boxed{0_3}$

$00_0 0 \quad H = 11\boxed{0_3}$

S_1

$0_1 10 \quad H = 01\boxed{0_3}$

$1_1 10 \quad H = 11\boxed{0_3}$

$01_1 0 \quad H = 11\boxed{0_3}$

S_2

$000_2 \quad H = 11\boxed{0_3}$

$010_2 \quad H = 11\boxed{0_3}$

$1_2 0 \quad H = 1\boxed{0_3}$

S_3

$H = 01\boxed{0_3}$

$H = 11\boxed{0_3}$

$H = 1\boxed{0_3}$

$H = \boxed{0_3}$

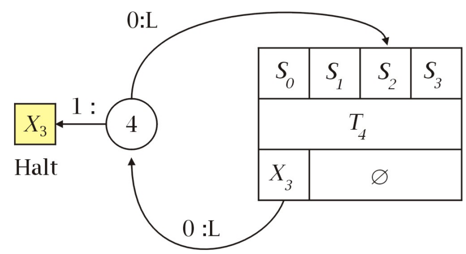


Figure 57.4. T5 for no. 57.

For T5 (*figure 57.4*)

Exit at X_3

1_4

$0_3 1$

$00_0 01$	$01_0 01$	$0_0 101$	$110_2 1$	$010_2 1$	$1_2 01$
END	END	END	$0:L$	$0:L$	$0:L$

NOT $S_0 = \overline{\overline{0}}$. Therefore, does not halt for $S_0 = \overline{\overline{0}}$.

Standard starting configuration: $S_0 = \overline{\overline{0_0}}$.

This machine does halt for $S_0 = 00_0 01$ but not for $S_0 = \overline{\overline{0_0}}$.

Hold out no. 58. Kellett B7.

No. 58 has cycles of length 1, 3 and 4 states, and direct solution presents difficulties (*figure 58.1*). We examine first the sub-machine T4 obtained by removing state Q4.

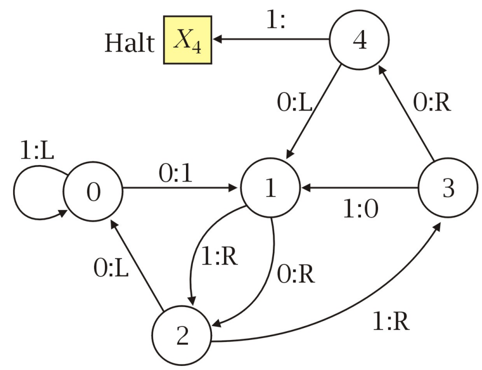


Figure 58.1. Hold out no. 57.

T4 (*figure 58.2*) halts in state Q3 on a 1. We solve by the method of exits (see subsequent two pages) to write the complete criterion. In the solution below, impossible configurations are shown in red. These denote termini to the branches generated by the method of exits. T4 does halt for standard configuration, and this is shown in green.

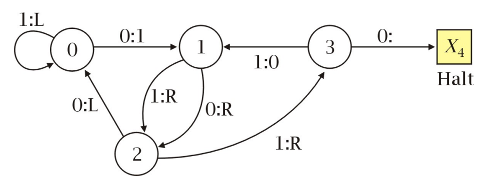


Figure 58.2 T4 for no. 57.

T4 is also a machine of some complexity, and the writing of its complete criterion is a task requiring patience. Looking at the structure of T4 we see several sub-cycles of length 1, 3 and 4. The sub-cycle of length 1 constitutes a trace through a finite string of 1s on the tape, exiting state Q0 only when the first 0 to the left of such a string is encountered. Because of the existence of 4 right moves as well as 1 more left move in the program, as well as the options to write 0 to 1 and 1 to 0, the machine in theory could loop through combinations of cycles.

For all these reasons it is essential to grasp the fundamental principle that lies behind the writing of a complete criterion. Because of the finite number of states and instructions in this machine program, whatever happens to this machine is completely determined by finite information, even where that finite information may have to be encoded by symbols representing finitely and infinitely repeating configurations of 1s and 0s on the Turing tape.

In the case of T4 it is essential to consider the possibility of loop configurations, but in practice the backward trace reveals that there are none. The tree generated by the method of exits is always finite.

Hold out no. 58. Kellett B7. *Continued.*

Tree for T4 generated by the method of exits.

LINE				
0	Exit at X_3			
1	0_3			
2	1_20			
3	1_110		0_110	
4			1_310	
5			1_2110	
6			1_11110	0_11110
8	01_00		01_0110	GO TO LINE 31
9	010_2		011_010	
10	01_10		0111_00	
11	00_00	*SC*	01110_2	
12	000_2		0111_10	
13	00_10		0110_00	
14	01_30		01100_2	
15	$1:R$		0110_10	
16			0111_30	
17			011_210	
18			01_1110	
19			00_0110	
20			001_010	
21			0011_00	
22			00110_2	
23			0011_10	
24			0010_00	
25			00100_2	
26			0010_10	
27			0011_30	
28			001_210	
29			00_1110	
30			01_3110	
			$1:R$	

LINE		
31	0_11110	
32	1_31110	
33	1_211110	
34	$1_1111110$	
35	$0_0111110$	
36	011111_00	$1:L$
37	0111110_2	START SUB-ROUTINE $1110_2 \to 1_3110$
38	011111_10	THIS SUB-ROUTINE
39	011110_00	ALSO IN LINES 12 TO 31
40	0111100_2	
41	011110_10	
42	011111_30	
43	01111_210	
44	0111_1110	
45	0110_0110	
46	011011_00	$1:L$
47	0110110_2	
48	011011_10	
49	011010_00	
50	0110100_2	
51	011010_10	
52	011011_30	
53	01101_210	
54	0110_1110	
55	0111_3110	END SUB-ROUTINE $1110_2 \to 1_3110$
56	011_21110	
57	$01_1111110$	
58	$00_0111110$	
59	0011111_00	$1:L$
60	00111110_2	
61	00111_3110	$1110_2 \to 1_3110$
62	0011_21110	
63	001_111110	
64	000_011110	
65	0001111_00	$1:L$
66	00011110_2	
67	00011_3110	$1110_2 \to 1_3110$
68	0001_21110	
69	000_111110	
70	001_311110	
	$1:R$	

Hold out no. 58. Kellett B7. *Continued.*

From the tree generated by the method of exits it is a straightforward matter to write the complete criterion for T4. Here we concentrate only on demonstrating that no. 58 does not halt.

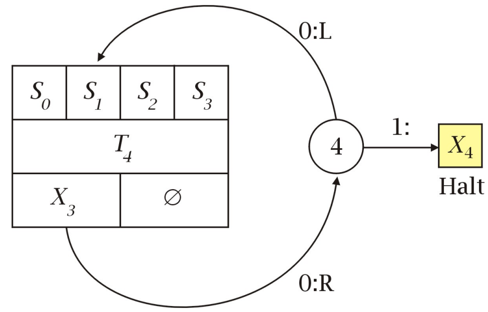


Figure 58.3. T5 for no. 57.

No. 58 may be viewed as the five-state machine, T5, obtained by addition of state Q4 to T4. In order to halt for standard configuration the machine must exit at X3, then loop through the stages X3, Q4, S1, X3 possibly several times before exiting at X3 in a configuration that causes it to halt at X4.

Because of the 0:L between Q4 and S1 any loop at S1 must contain the configuration $0_1 0$. For an input of this configuration to exit again at X3 we see from the tree for T4, the only possibilities are:

LINE 13	$0110_1 0$
LINE 15	$00_1 0$
LINE 26	$0010_1 0$
LINE 41	$011110_1 0$
LINE 51	$011010_1 0$

Consider the input of standard starting configuration at *S0*.

Standard starting configuration: $S_0 = \overline{\overline{0_0}}$.

$S_0 = \overline{\overline{0_0}}$
$X_3 = 110_3$
1100_4
$S_1 = 110_1 0$
$X_3 = 1110_3$
11100_4
$1110_1 0$
$X_3 = 11100_3$
111000_4
$S_1 = 11100_1 0$

This last configuration is not a halting configuration for T4. Therefore T4 = no. 58 does not halt.

Hold out no. 59. Kellett B8.

No. 59 has cycles of length 1, 2, 3 and 4 states, and direct solution, though possible, presents difficulties (*figure 59.1*). We examine first the sub-machine T4 obtained by removing state Q4 (*figure 59.2*).

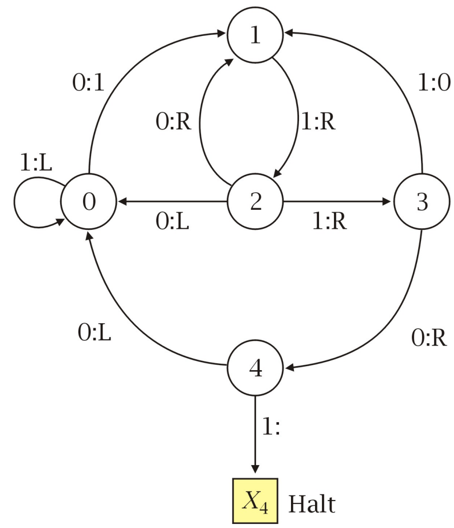


Figure 59.1. Hold out no. 59.

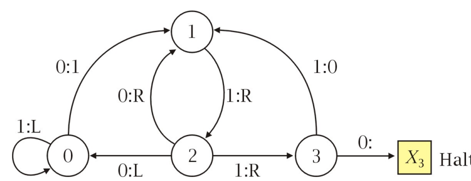


Figure 59.2. T4 for no. 59.

T4 halts in state Q3 on a 1. We solve by the method of exits to write the complete criterion. In the solution below, impossible configurations are shown in red. These denote termini to the branches generated by the method of exits. Loop configurations are shown in blue. T4 does halt for standard configuration, and this is shown in green.

Exit at X_3					
0_3					
$1_2 0$					
$0_1 10$				$1_1 10$	
$1_3 10$	$0:1$			$0_0 10$	
$1_2 110$				$01_0 0$	$1:L$
$0_1 1110$		$1_1 1110$		010_2	
$1_3 1110$	$0:1$	$0_0 1110$		$01_1 0$	$0:R$
$1_2 11110$		$0111_1 0$		$00_0 0$	$S_0 = \bar{\bar{0}}$
$0_1 1\overline{11}0$		$0110_0 0$		000_2	
$1_3 1\overline{11}0$		01100_2		$00_1 0$	$1:R$
$1_2 \overline{11}0$		$0110_1 0$	$1:R$	$01_3 0$	
		$0111_3 0$		$1:R$	
		$01_1 110$			
		$01_1 110$			
		$00_0 110$			
		$0011_0 0$	$1:L$		
		00110_2			
		$0011_1 0$			
		$0010_0 0$			
		$1:L$			

Hold out no. 59. Kellett B8. *Continued.*

Complete criterion for T4

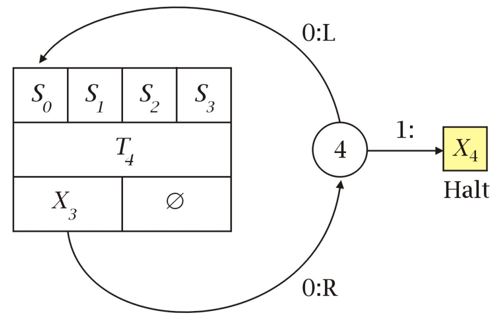


Figure 59.3. T5 for no. 59.

S_0		S_1		S_2	
$0_0 1110$	$H = 1101\boxed{0_3}$	$0_1 10$	$H = 01\boxed{0_3}$	$1_2 0$	$H = 1\boxed{0_3}$
$01_0 110$	$H = 1101\boxed{0_3}$	$0_1 1110$	$H = 1101\boxed{0_3}$	$1_2 110$	$H = 101\boxed{0_3}$
$01_0 110$	$H = 1101\boxed{0_3}$	$1_1 1110$	$H = 1101\boxed{0_3}$	$1_2 \overline{11}0$	$H = 1\overline{01}\boxed{0_3}$
$011_0 10$	$H = 1101\boxed{0_3}$	$0111_1 0$	$H = 1101\boxed{0_3}$	01100_2	$H = 1101\boxed{0_3}$
$0111_0 0$	$H = 1101\boxed{0_3}$	$0110_1 0$	$H = 1101\boxed{0_3}$	$011_2 10$	$H = 1101\boxed{0_3}$
$0110_0 0$	$H = 1101\boxed{0_3}$	$01_1 110$	$H = 1101\boxed{0_3}$	00110_2	$H = 1101\boxed{0_3}$
$00_0 110$	$H = 1101\boxed{0_3}$	$0011_1 0$	$H = 1101\boxed{0_3}$	010_2	$H = 11\boxed{0_3}$
$001_0 10$	$H = 1101\boxed{0_3}$	$01_1 \overline{11}0$	$H = 1\overline{10}1\boxed{0_3}$	000_2	$H = 11\boxed{0_3}$
$0011_0 0$	$H = 1101\boxed{0_3}$	$1_1 10$	$H = 11\boxed{0_3}$	S_3	
$0010_0 0$	$H = 1101\boxed{0_3}$	$01_1 0$	$H = 11\boxed{0_3}$	$1_3 1\overline{11}0$	$H = 1\overline{10}1\boxed{0_3}$
$0_0 10$	$H = 11\boxed{0_3}$	$00_1 0$	$H = 11\boxed{0_3}$	$1_3 10$	$H = 01\boxed{0_3}$
$01_0 0$	$H = 11\boxed{0_3}$			$0111_3 0$	$H = 1101\boxed{0_3}$
$00_0 0$	$H = 11\boxed{0_3}$			$01_3 0$	$H = 11\boxed{0_3}$

From the complete criterion for T4 we can now prove that T5 = no. 59 does not halt for standard configuration. This is done by considering the effect of input of standard starting configuration on T5 (*figure 59.3*).

$S_0 = \overline{\overline{0_0}}$ Standard starting configuration
$X_3 = 110_3$
1100_4
$S_0 = 110_0 0$ First loop to T_4. Halting for T_4
$X_3 = 11010_3$
110100_4
$S_0 = 11010_0 0$ Second loop to T_4. Non-halting for T_4
T_5 never returns to X_3 and does not halt for standard configuration.

Hold out no. 61. Kellett A83.

No. 61 has two exits, X3 and X4 (*figure 61.1*). Solution directly is tricky, because it contains loops of every possible length—1,2,3,4 and 5-cycles; hence, there are many permutations. We solve by considering the sub-machine, T4, obtained by removing state Q0.

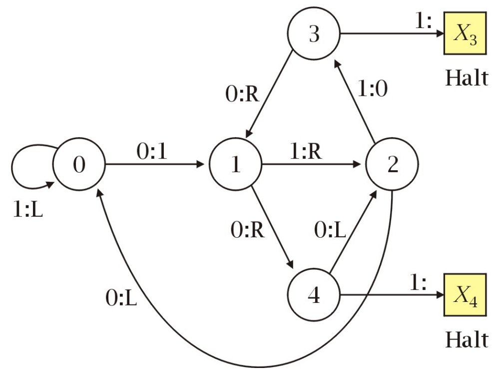


Figure 61.1. Hold out no. 61.

T4 has three exits at X2, X3 and X4 (*figure 61.2*). We solve by the method of exits to write the complete criterion. In the solution below, an impossible configuration is shown in red. These denote termini to the branches of the tree generated by the method of exits. Loop configurations are shown in blue, which also denote termini of the tree.

Exit at X_2				Exit at X_3	Exit at X_4	
0_2						
$1_1 0$		00_4		1_3	1_4	
$0_3 10$		$0_1 0$		$1:0$	$0_1 1$	
$1_2 10$		$0_3 00$			$0_3 01$	
$1_1 110$	$0:L$	$1_2 00$			$1_2 01$	
$0_3 1110$		$1_1 100$	$10_4 0$		$1_1 101$	$10_4 1$
$1_2 1110$		$0_3 1100$	$0:R$		$0_3 1101$	$0:R$
$1_2 \overline{11}10$		$1_2 1100$			$1_2 1101$	
$1_1 \overline{11}0$		$1_1 11100$	$0:L$		$1_1 11101$	$0:L$
$0_3 \overline{11}10$		$1_1 \overline{11}100$			$1_1 \overline{11}101$	
		$1_2 \overline{11}00$			$1_2 \overline{11}01$	
		$0_3 \overline{11}00$			$0_3 \overline{11}01$	

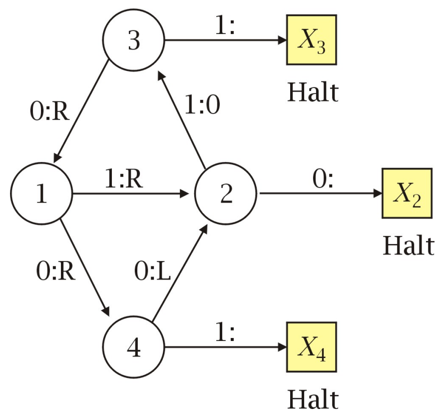


Figure 61.2. T4 for no. 61.

Complete criterion for T4

S_1		S_2	
$1_1 101$	$H = 110\boxed{1_4}$	$1_2 01$	$H = 10\boxed{1_4}$
$1_1 0$	$H = 1\boxed{0_2}$	0_2	$H = \boxed{0_2}$
$1_1 \overline{11}0$	$H = 0\overline{10}1\boxed{0_2}$	$1_2 \overline{11}10$	$H = 0\overline{00}1\boxed{0_2}$
$1_1 \overline{11}100$	$H = 0\overline{00}\boxed{0_2}0$	$1_2 \overline{11}00$	$H = 0\overline{00}0\boxed{0_2}$
$1_1 \overline{11}101$	$H = 0\overline{01}00\boxed{1_4}$	$1_2 \overline{11}01$	$H = 0\overline{00}0\boxed{1_4}$

S_3		S_4	
1_3	$H = \boxed{1_3}$	$10_4 1$	$H = 00\boxed{1_4}$
$0_3 10$	$H = 01\boxed{0_2}$	1_4	$H = \boxed{1_4}$
$0_3 \overline{11}10$	$H = 0\overline{01}\boxed{0_2}$	00_4	$H = \boxed{0_2}0$
$0_3 \overline{11}01$	$H = 0\overline{01}0\boxed{1_4}$	$10_4 0$	$H = 0\boxed{0_2}0$

Hold out no. 61. Kellett A83. *Continued.*

We obtain T5 by adding the state Q0 to T4 (*Figure 61.3*).

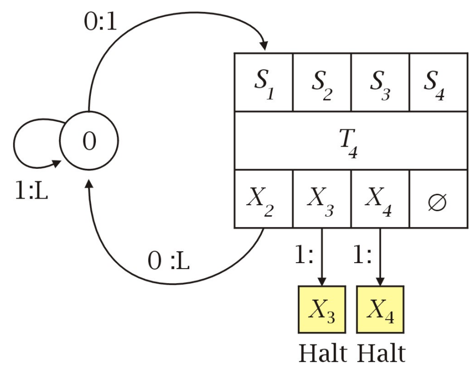


Figure 61.3. T4 for no. 61.

Standard starting configuration: $S_0 = \overline{\overline{0}}_0$.

Only if there is an input at S3 can T5 exit at X3. But an input of the standard starting configuration at S0 cannot lead to an input at S3. Therefore T5 cannot halt at X3.

Suppose T5 can exit at X4 by starting in standard starting configuration at Q0. Since T5 cannot loop back to Q0 by S2, S3 or S4 the only possible input is at S1.

$$1_1\,\overline{11}101 \qquad H = 0\,\overline{01}\,0\boxed{1_4}$$
$$0_0\,\overline{11}101 \qquad 0:L$$
$$0\overline{11}1_0 01$$
$$0\overline{11}10_2 1$$
$$0\overline{11}1_1 01 \qquad 0:L$$
$$0\overline{11}\,0_0 01 \qquad 0:L$$

The only possible halting configurations for an input at S0 are:

$$0_0\,\overline{11}101$$
$$0\overline{11}1_0 01$$
$$0\overline{11}\,0_0 0 \qquad H = 0\,\overline{01}\,00\boxed{1_4}$$

None of these are standard configuration. Therefore T5 does not halt for standard configuration.

Hold out no. 63. Kellett A84.

No. 63 has one exit at X4 (*figure 63.1*). Solution directly is tricky, because of the three inputs to state Q2, which cause many branches in the tree generated by the method of exits. It contains loops of every possible length—1,2,3,4 and 5-cycles; hence, there are many permutations. We solve by considering the sub-machine, T3, obtained by removing states Q0 and Q2.

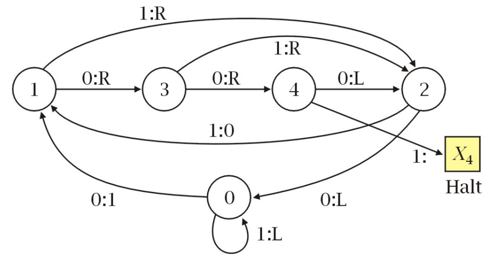


Figure 63.1. Hold out no. 63.

T3 has three exits at X1, X3 and X4 (*figure 63.2*). We solve by the method of exits to write the complete criterion. In the solution, an impossible configuration is shown in red. These denote termini to the branches of the tree generated by the method of exits.

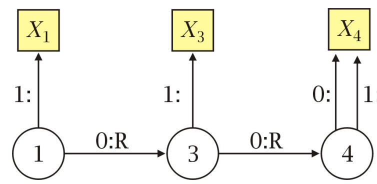


Figure 63.2. T3 for no. 63.

Exit at X_1

$1_1 \quad H = \boxed{1_1}$

Exit at X_3

$1_3 \quad H = \boxed{1_3}$

$0_1 1 \quad H = 0\boxed{1_3}$

Exit at X_4

$1_4 \quad H = \boxed{1_4}$	$0_4 \quad H = \boxed{0_4}$
$0_3 1 \quad H = 0\boxed{1_4}$	$0_3 0 \quad H = 0\boxed{0_4}$
$0_1 01 \quad H = 00\boxed{1_4}$	$0_1 00 \quad H = 00\boxed{0_4}$

Complete criterion for T3

S_1		S_3		S_4	
1_1	$H = \boxed{1_1}$	1_3	$H = \boxed{1_3}$	0_4	$H = \boxed{0_4}$
$0_1 1$	$H = 0\boxed{1_3}$	$0_3 1$	$H = 0\boxed{1_4}$	1_4	$H = \boxed{1_4}$
$0_1 01$	$H = 00\boxed{1_4}$	$0_3 0$	$H = 0\boxed{0_4}$		
$0_1 00$	$H = 00\boxed{0_4}$				

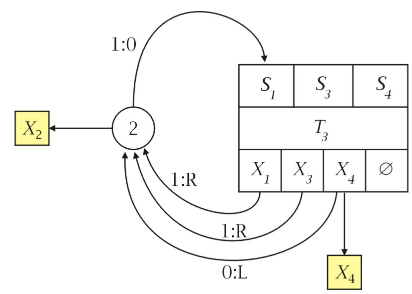


Figure 63.3. T4 for no. 63.

We obtain T4 by adding the state Q2 to T3 (*figure 63.3*), and apply the method of exits. In the solution, an impossible configuration is shown in red. These denote termini to the branches of the tree generated by the method of exits. Certain configurations cannot be traced further backwards; these are marked in red by END. Loop configurations are generated.

Hold out no. 63. Kellett A84. *Continued.*

Exit at X_4

1_4				
$0_3 1$	$0_1 01$			
END	$1_2 01$			
	$1_1 101$	$1_3 101$	$10_4 1$	
	$1:0$	$1_2 1101$	END	
		$1_1 11101$	$1_3 11101$	$0:L$
		$1:0$	$0_1 111101$	

Loops: $1_2\overline{11}01 \quad H = 0\overline{01}0\boxed{1_4}$

$1_3\overline{11}101 \quad H = 0\overline{01}00\boxed{1_4}$

$0_1\overline{11}101 \quad H = 1\overline{01}00\boxed{1_4}$

$1_1\overline{11}101 \quad H = 1\overline{01}00\boxed{1_4}$

Exit at X_2

0_2								
$1_1 0$	$1_3 0$				00_4			
$1:0$	$0_1 10$				$0_1 00$	$0_3 0$		
	$1_2 10$				$1_2 00$			
	$1_1 110$	$1_3 110$	$0:L$		$1_1 100$	$1_3 100$	100_4	
	$1:0$	$0_1 1110$			$1:0$	$0_1 1100$	END	
		$1_2 1110$				$1_2 1100$		
		$1_1 11110$	$1_3 11110$	$0:L$		$1_1 11100$	$1_3 11100$	$0:L$
							$0_3 111100$	
							$1_2 111100$	

Loops

$0_1\overline{11}10 \quad H = 0\overline{10}1\boxed{0_2}$

$1_2\overline{11}10 \quad H = 0\overline{10}1\boxed{0_2}$

$1_3\overline{11}0 \quad H = 0\overline{10}1\boxed{0_2}$

$1_1\overline{11}0 \quad H = 1\overline{01}1\boxed{0_2}$

$1_3\overline{11}100 \quad H = 1\overline{01}\boxed{0_2}00$

$1_2\overline{11}00 \quad H = 0\overline{10}\boxed{0_2}0$

$0_1\overline{11}00 \quad H = 0\overline{10}1\boxed{0_2}$

$1_1\overline{11}00 \quad H = 0\overline{10}1\boxed{0_2}$

Hold out no. 63. Kellett A84. *Continued.*

From this we could write the complete criterion for *T4*, and the find the complete criterion for *T5* which is obtained by adjoining state *Q0* to *T4*. However, since our objective is to demonstrate that that *T5* does not halt for the standard configuration, we will omit these details.

Consider the standard starting input: $S_0 = \overline{\overline{0_0}}$. The effect of this is that the scanned first 0 is converted to a 1, and the input is sent to *S1*. So we have: $S_1 = \bar{\bar{0}}1_1\bar{\bar{0}}$. At *S1* the tape configurations that lead to an exit are:

$$S_1 = \quad 0_1 01 \quad 1_1 101 \quad 0_1 \overline{1}1101 \quad 1_1 \overline{1}1101 \quad 1_1 0 \quad 0_1 \overline{1}110 \quad 1_1 \overline{1}10 \quad 1_1 \overline{1}100$$

Of these the only possible exit for the given input at *Q1* is $S_1 = 1_1 0$. But given the initial configuration this is equivalent to the input at *S2* of $S_2 = 010_2 00$. Hence

S_2
$010_2 0$
$01_0 00$
$0_0 100$
$1_1 100$

So, at the second visit to *Q1* the input is $S_1 = 1_1 100$. But considering the list of inputs at *S1* that lead to an exit, this configuration is not on the list. Hence, *T5* = no. 63 does not halt for the standard configuration.

Hold out no. 68. Kellett B10.

The presence of the 5-cycle and the many 1-cycles in no. 68 (*figure 68.1*) may give the impression that the halting problem for it is difficult. However, it is straightforward to demonstrate that no. 68 does not halt for standard configuration. It has only one exit at X4, where it halts on a 0. A backward trace by the method of exits constructs a tree with no branches and minimal depth.

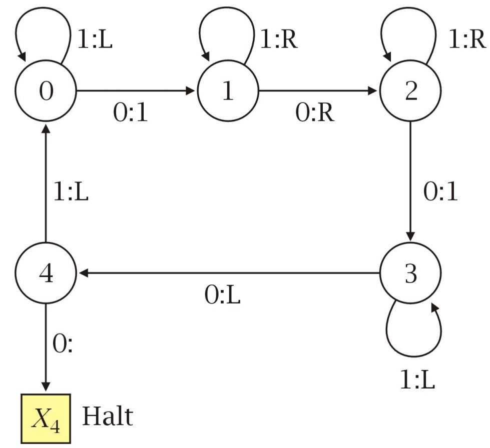


Figure 68.1. Hold out no. 68.

Exit at X_4

0_4

00_3

$00\overline{1_3}$

$00\bar{1}0_2$

$001_2\,\bar{1}0$

$00_1 1\bar{1}0$

$0:1$

$1:R$

There is no path backwards from this exit to the standard configuration. Therefore no. 68 does not halt for standard starting configuration.

Standard starting configuration: $S_0 = \overline{\overline{0_0}}$.

Hold out no. 69. Kellett B11.

No. 69 may be regarded as a machine of four states T4 obtained by removal of state Q4 to which Q4 has been added (*figure 69.1*). The point of this observation is that T4 only ever moves to the right; and only by addition of state Q4 does T5 = no. 69 gain the possibility of moving left.

No 69 halts in state Q3 on a 1. We solve by the method of exits. In the solution below, impossible configurations are shown in red. These denote termini to the branches generated by the method of exits. T4 has a sub-routine between states Q0, Q1 and Q2 and loops infinitely between them. It has loop configurations.

There is no path from the exit at X3 in the backward trace of no. 69 to the standard configuration, nor does this machine ever have a halting configuration in which Q4 appears. This means that the halting configurations for no. 69 and its T4 sub-machine are identical. After input of standard starting configuration at S1 machine no. 69 reaches state Q4 after five moves. Therefore, no, 69 does not halt for standard configuration.

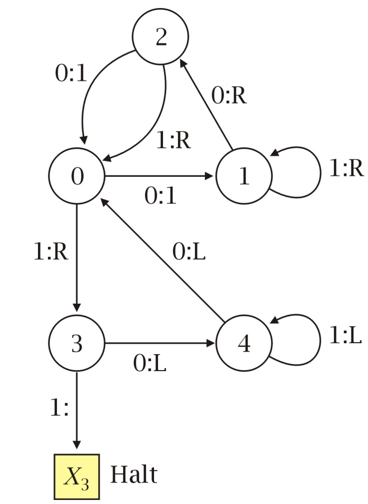


Figure 69.1. Hold out no. 69.

Standard starting configuration: $S_0 = \overline{\overline{0_0}}$.

Exit at X_3

1_3

$1_0 1$

$0_2 1$

$0_1 01$ START SUB-ROUTINE

$\overline{1_1}\,001$ $0_1 \to 0_1 10\bar{1}\,0$

$0_0\overline{1_1}\,001$ $0:1,\ 0:L$

$0_1\overline{10\bar{1}}\,001$

$1_2\overline{0\bar{1}}\,001$

$1_2 0\bar{1}\,001$

$0_1 10\bar{1}\,001$

$\overline{1_1}\,010\bar{1}\,001$

$0_0\bar{1}\,010\bar{1}\,001$

$1_2 0\bar{1}\,010\bar{1}\,001$

LOOP

$0_0\overline{\bar{1}\,010}\bar{1}\,001$

$0_1\overline{10\bar{1}}\,001$

$1_2\overline{0\bar{1}}\,001$

$1_2 11$

$1_2 0\bar{1}\,0111$ SUB-ROUTINE: $1_2 \to 1_2 0\bar{1}\,00$

LOOP

$1_2\overline{0\bar{1}\,0}011$

$0_1\overline{10\bar{1}}\,011$

$0_0\overline{\bar{1}\,010}\bar{1}\,0111$

$S_0 = \overline{\overline{0_0}}$

1_1

10_1

100_2

1010_3

$001_4 0$

Hold out no. 71. Kellett A85.

No. 71 has one exit at X3 (*figure 71.1*). We solve by considering the sub-machine, T4, obtained by removing state Q3 (*figure 71.2*). T4 has two exits at X2 and X4. We solve by the method of exits to write the complete criterion. In the solution, an impossible configuration is shown in red. These denote termini to the branches of the tree generated by the method of exits. Certain configurations cannot be traced further backwards; these are marked in red by END.

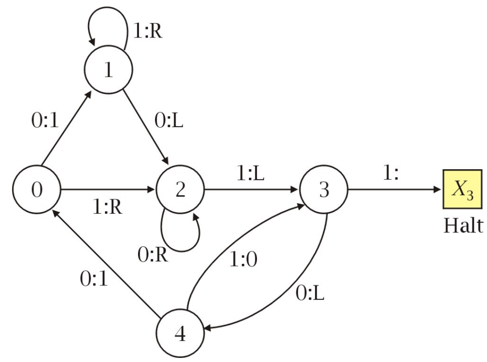


Figure 71.1. Hold out no. 71.

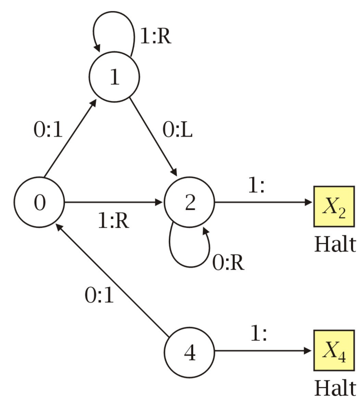


Figure 71.2. T4 for no. 71.

Exit at X_4	Exit at X_2						
1_4	1_2						
	$0_2 1$			$1_0 1$	$10_3 1$		
	$0:1$	$0_2 \bar{0} 1$	$1_0 \bar{0} 1$	$0_4 1$	$0:1$	$1_2 0$	
		END	$0_4 \bar{0} 1$	END		$0_0 0$	$1_1 \bar{1} 0$
			END			END	$0_0 \bar{1} 0$
							$0:1$

S_0		S_1	
$0_0 0$	$H = 1_2 0$	10_3	$H = 1_2 0$
$0_0 \bar{1} 0$	$H = 1\bar{1}1_2 0$	$1_1 0$	$H = 1_2 0$
		$1_1 \bar{1} 0$	$H = 1\bar{1}1_2 0$

S_3		S_4	
1_2	$H = 1_2$	$0_4 1$	$H = 1_2 1$
$0_2 1$	$H = 01_2$	$0_4 \bar{0} 1$	$H = 1\bar{0}1_2$
$0\bar{0}1$	$H = 0\bar{0}1_2$	1_4	$H = 1_4$

We obtain T5 by adding the state Q3 to T4, and apply the method of exits. The machine can only halt if there is an path from the standard starting configuration at S0, or if there is an input at S4, so inputs at S1 and S2 cannot lead to a standard configuration and may be ignored. In the tree of exits below impossible configurations are shown in red.

Standard starting configuration: $S_0 = \overline{\overline{0_0}}$.

Exit at X_3		S_0		S_4		
1_3		$0_0 0$	$0_0 \bar{1} 0$	$0_4 1$	$0_4 \bar{0} 1$	
$1:0$	11_2			$0:L$	$00_3 \bar{0} 1$	
					$1:L$	$01_4 \bar{0} 1$
						END

In order to exit and halt at X3 there must be an input at S0 or S4. Any input at S0 is impossible. None of the possible inputs at S4 lead backwards to the standard configuration. Therefore, T5 = Holdout 71 does not halt for standard starting configuration.

Hold out no. 74. Kellett B12.

No. 74 has many sub-cycles which complicate the process of finding a direct solution (*figure 74.1*). We therefore solve by investigating the halting behaviour of the submachine T4 obtained by removing state Q4 (*figure 74.2.*).

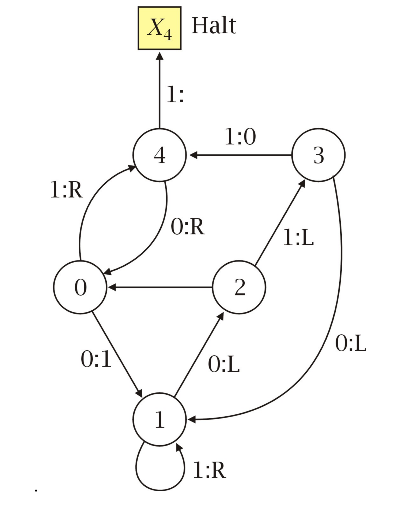


Figure 74.1. Hold out no. 74.

T4 has a sub-routine initiated whenever it enters state Q3 scanning a 0. The computation of this sub-routine is shown to the left. To return to the same state scanning a 0 the tape must be configured as shown. In the summary, any permutation of finite blocks of 01 or 010 will return the machine to state Q3 scanning a 0. We solve for T4 by the method of exits. In the solution below, impossible configurations are shown in red. These denote termini to the branches generated by the method of exits. Because of the sub-routine T4 has loop configurations. It does halt (exit at X0) for standard configuration, shown in green.

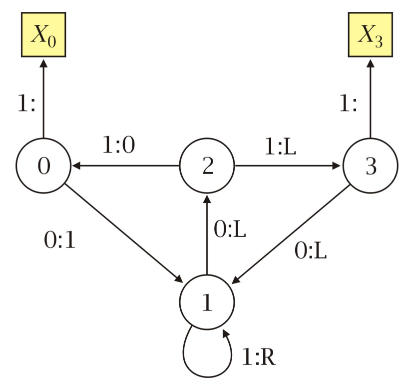


Figure 74.2. T4 for no. 74.

Sub-routine (backwards by method of exits)

0_3

01_2

010_1

01_10 $\quad\quad$ 0100_3

00_00 $\quad$ 010_3 $\quad\quad$ $0_3 \to 0100_3$

$0:1$ $\quad$ $0_3 \to 010_3$

$$Summary: \; 0_3 \to \begin{Bmatrix} \overline{01} \\ \overline{010} \end{Bmatrix} 0_3$$

Exit at X_0

1_0

0_2

000_3 $\quad$ $1:R$

0001_2

00010_1

0001_10 $\quad\quad$ 000100_3

0000_00 $\quad SC$ $\quad\quad$ $0_3 \to \begin{Bmatrix} \overline{01} \\ \overline{010} \end{Bmatrix} 0_3$

00010_3

$$0_3 \to \begin{Bmatrix} \overline{01} \\ \overline{010} \end{Bmatrix} 0_3$$

$$Summary: \; 00 \begin{Bmatrix} \overline{01} \\ \overline{010} \end{Bmatrix} \begin{cases} 0_3 \\ 01_2 \\ 010_1 \\ 01_10 \end{cases}$$

Hold out no. 74. Kellett B12. *Continued*

Exit at X_3

1_0

11_3

110_1

11_10 $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $1100_3 \quad 0:1$

$10_00 \quad 1_110 \quad\quad 110_3 \quad\quad 0_3 \rightarrow \begin{Bmatrix} \overline{01} \\ \overline{010} \end{Bmatrix} 0_3$

$0:1 \quad \overline{1}_1 110 \quad\quad 0_3 \rightarrow \begin{Bmatrix} \overline{01} \\ \overline{010} \end{Bmatrix} 0_3$

$0_0\overline{1}_1110 \; 0:L$

$0:1$

$$\textit{Summary}: \left.\begin{matrix} 1 \\ 11 \\ 110 \\ 100 \\ 110 \\ 1\overline{1}110 \\ 0\overline{1}110 \end{matrix}\right\} \begin{Bmatrix} \overline{01} \\ \overline{010} \end{Bmatrix} \left\{\begin{matrix} 0_3 \\ 01_2 \\ 010_1 \\ 01_10 \end{matrix}\right.$$

T5 is formed by the addition of state Q4 to T4 (*figure 74.3*). In order to halt for standard starting configuration, T5 must pass through the feedback loop S0, X0 or X3, Q4, S0 at least once. Hence, in the complete criterion for T4 only the input at S0 needs to be considered. We can of course from the tree generated by the method of exits for T4 write its complete criterion, but here we focus on solving the halting problem for T5 only.

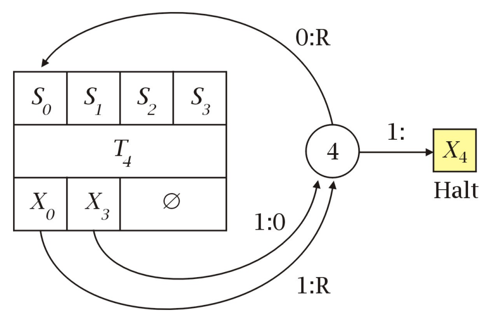


Figure 74.3. T5 for no. 74.

S_0	
1_0	$H = \boxed{1_0}$
0000_00	$H = \boxed{1_0}0010$
10_00	$H = \boxed{1_3}10$
$0_0\overline{1}110$	$H = 11\overline{1}\boxed{1_3}10$

Non-halting behaviour of T5

$S_0 = \overline{\overline{0}}_0$

$X_0 = 1_00010$

10_4010

$S_0 = 100_010$

Not a halting input for S_0. No exit to X_0 or X_3.

Standard starting configuration: $S_0 = \overline{\overline{0}}_0$.

We see that even after the first loop back to *S0* we have a non-halting configuration at *S0.* Therefore, *T5* = no. 74 does not halt for standard configuration.

Hold out no. 87. Kellett B13.

No. 87 has many sub-cycles which complicate the process of finding a direct solution (*figure 87.1*). We therefore solve by investigating the halting behaviour of the submachine T4 obtained by removing state Q1. As a heuristic—this would seem to be a good choice of initial state to remove, as it simplifies the problem by removing the infinite 1:R loop.

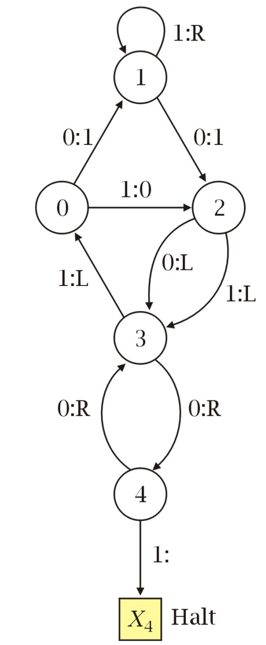


Figure 87.1. Hold out no. 87.

T4 has exits at X0 and X4. We solve for T4 by the method of exits. In the solution below, impossible configurations are shown in red. These denote termini to the branches generated by the method of exits. T4 has loop configurations. T4 halts on standard configuration: in fact, it exits immediately at Q0. This exit is shown in green in the tree diagram below.

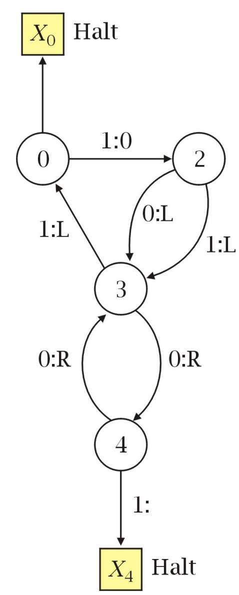


Figure 87.2. T4 for no. 87.

Exit at X_0

0_0	SC						
01_3							
0_41					011_2	010_2	
0_301					$0:1$	011_0	
0_4001			00_21	$1:L$		0111_3	
0_30001			01_01			01110_2	01111_2
0_400001	$1:L$	00_2001	011_3			01111_0	$1:0$
$0_3000001$			01_0001	0111_2	0110_2	$0:R$	LOOP
LOOP		$1:L$	$1:0$	0111_0			
				01111_3			
				LOOP			

Summary: exit configurations

$$\left.\begin{matrix}0_40\\0_3\\00_20\\01_00\end{matrix}\right\}\overline{00}\,01 \qquad 01\overline{11}\begin{cases}11_0\\10_2\\1_3\end{cases} \qquad 01\overline{11}\begin{cases}1_0\\0_21\\11_3\end{cases}$$

$0_0,\ 01_3,\ 01_01,\ 011_3,\ 0111_2, 011_2,\ 010_2,\ 011_0,\ 0111_3$

Exit at X_4

1_4				
0_31				
0_401			01_2	$0:L$
0_3001			$1:0$	
0_40001	$1:L$	00_201		
0_300001		01_001		
LOOP		$1:L$		

Summary: exit at X_4

$$\left.\begin{matrix}0_4\\0_30\\00_2\\01_0\end{matrix}\right\}\overline{00}\,01 \qquad 01_2,\ 0_31,\ 1_4$$

Hold out no. 87. Kellett B13. *Continued*

T5 is formed from T4 by the addition of state Q1 (*figure 87.3*). From the tree generated by the method of exits for T4 we could write a complete criterion for T4, but as here we are concerned to demonstrate that T5 does not halt, we consider only the input at S2 and write a complete criterion for that.

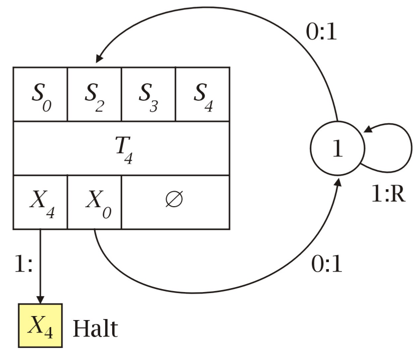


Figure 87.3. T5 for no. 87.

S_2

010_2	$H = \boxed{0_0}10$
011_2	$H = \boxed{0_0}11$
0111_2	$H = 0\boxed{0_0}11$
$01\overline{11}0_21$	$H = \boxed{0_0}1\overline{01}01$
$01\overline{11}10_2$	$H = 0\boxed{0_0}\overline{01}01$
$00_20\overline{00}01$	$H = 000\overline{00}\boxed{0_0}1$
01_2	$0\boxed{1_4}$
$00_2\overline{00}01$	$H = 00\overline{00}0\boxed{1_4}$

Non-halting behaviour of T5

Standard starting configuration: $S_0 = \overline{\overline{0_0}}$.

$S_0 = \overline{\overline{0}}$

$X_0 = \overline{\overline{0}}1_1\overline{\overline{0}}$

$\overline{\overline{0}}10_1\overline{\overline{0}}$

$S_2 = \overline{\overline{0}}11_2\overline{\overline{0}}$

$X_0 = \overline{\overline{0}}0_011\overline{\overline{0}}$

$\overline{\overline{0}}1_111\overline{\overline{0}}$

$\overline{\overline{0}}1110_1\overline{\overline{0}}$

$S_2 = \overline{\overline{0}}1111_2\overline{\overline{0}}$

Not a halting input for S_2. No exit to X_0 or X_4.

After the second loop back to S0 we have a non-halting configuration at S2. Therefore, T5 = no. 87 does not halt for standard configuration.

Notes to the solution of other hold outs

The machines referred to here as hold outs are from Kellett's [2005] paper. We do not present the solutions in the same detail as in the previous chapter. The same "caveat emptor" applies. The soundness of the method is established in the text, but the individual solutions, which have been worked by hand, are at this time of writing unchecked by any independent body. We claim that subject to this checking, all 98 hold outs are non-halting, and hence the productivity of the five-state Turing machines is 11, which is the productivity of the B5 Champion.

We provide notes as to the solutions. The iterative method described in the text (*Solution to the Halting Problem*) may be deployed by subtracting any single state from the given machine to obtain a four-state machine designated *T4*; this process may be repeated to obtain a three-state machine, and so forth. The state subtracted is designated *Qi,* $0 \le i \le 4$. In most cases the author proceeded by making the four-state machine that from inspection seemed most likely to render the problem more tractable.

Hold out no. 0

$T4 = T5 - Q4$. $T4$ has three exits, X_0, X_2, X_3. Construction of the tree by the method of exits, enables the complete criterion for $T4$ to be written. There is a sub-routine.

$$\left.\begin{array}{l} 1_1 0 \\ 0_0 0 \\ 00_1 \\ 0_2 1 \end{array}\right\} \bar{1}0$$

$T5$ can be shown to be non-halting by observing the effect of the input of the standard starting configuration, given the complete criterion for $T4$.

S_0	$\overline{\overline{0_0}}$	
X_3	100_3	
	1000_4	
	$1_4 000$	
S_1	$0_1 1000$	
X_3	$1100_3 0$	sub-routine: $1100_3 0 \to 11100_3 0$
S_1	$0_1 11000$	
X_3	$11100_3 0$	
	LOOP	

Hold out no. 1

$T4 = T5 - Q3$. The $T5$ machine has an exit at X_3. In order to halt (exit at X_3), the path taken has the form

$$S_0 \to X_2 \to Q_3 \to \underbrace{\boxed{S_4 \to X_2}}_{\text{REPEATED}} \to X_3 \to \text{HALT}$$

So, when we apply the method of exits to $T4$ and write its complete criterion, it is only the inputs at S_4 that are critical. The method of exits gives the criterion of S_4 as

S_4	
10_4	$H = 10_2$
$\overline{1_4}10$	$H = \bar{1}10_2$
000_4	$H = 11_20$

Then, when in $T5$ we examine the input of the standard starting configuration we obtain

S_0	$\overline{\overline{0_0}}$	
X_3	10_2	
	100_3	
S_4	1000_4	From the criterion, $000_4 \to H = 11_20$
X_2	111_20	
	110_30	
	1100_4	

This last configuration is non-halting for S_4. Hence, $T5$ does not halt.

Hold out no. 2

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_3.

S_3

1_3	$H = 1_3$
00_31	$H = 1_111$
10_31	$H = 1_111$
0_30	$H = 01_1$

Apply the method of exits to $T5$.

exit at X_2

1_2				
$1:0$	1_11			
	1_111 required (RQ)			
	00_31	10_31		
	0_201	$0:R$		
	1_101		1_3001	
	01_101 RQ		0_21001	
	0_3001		1_31001	1_101001
	0_20001		LOOP	LOOP
	1_100001	1_30001		
	LOOP	LOOP		

The halting configurations are

$$\left.\begin{matrix} 0_2 \\ 01_1 \\ 0_30 \\ 1_30 \end{matrix}\right\} \left\{\begin{matrix} \overline{00} \\ \overline{10} \end{matrix}\right\} 01$$

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 2 does not halt for standard starting configuration.

Hold out no. 3

$T4 = T5 - Q4$. *T*4 has exits at X_0, X_2, X_3. Apply method of exits and write the complete criterion for *T*4. There is a sub-routine. Then by examination of the input of standard starting configuration on *T*5, we obtain

LINE			
1	S_0	$\overline{\overline{0}}_0$	
2	X_3	100_3	
3		$10_4 0$	
4		$1_4 00$	
5	S_1	$0_1 100$	from the complete criterion for $T4$, $00_1 10 \to 1100_3$
6	X_3	$1100_3 0$	
7		$110_4 00$	
8		$11_4 000$	
9	S_1	$1_1 1000$	from the complete criterion for $T4$, $1_1 1 \to 11_2$
10	X_2	$11_2 000$	
11		$1_4 1000$	
12	S_1	$0_1 11000$	from the complete criterion for $T4$, $00_1 \bar{1} 0 \to \bar{1} 00_3$
13	X_3	$1100_3 000$	
		LOOP	compare with line 6

Described as a leaning Christmas Tree by Kellett [2005], it enters an infinite loop and does not halt. This can be detected also from the trace.

Hold out no. 5

$T4 = T5 - Q3$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_1 and S_4.

S_1		S_4	
01_1	$H = 0_2 1$	0_4	$H = 0_4$
00_1	$H = 0_2 0$	010_4	$H = 0_4 10$
010_1	$H = 0_2 00$	$0\overline{10}10_4$	$H = 0_4 \overline{10}$
011_1	$H = 0_2 01$		

Apply the method of exits to $T5$.

exit at X_4			
0_4			
0_4			$0\overline{10}10_4$
00_3			$0\overline{10}100_3$
$0_2 0$			$0\overline{10}10_2 0$
00_1	010_1	011_1	SUB-ROUTINE
001_3	0101_3	0111_3	LOOP
$00_2 1$	$010_2 1$	$0 : R$	
001_1	0101_1		
0011_3	01011_3		
$0 : R$	$0 : R$		

The sub-routine gives halting configurations

$$\overline{0\overline{10}1}\begin{cases} 0_2 0 \\ 0011_3 \\ 01011_3 \\ 011_1 \\ \text{others} \end{cases}$$

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, $T5$ = no. 5 does not halt for standard starting configuration.

Hold out no. 6

$T4 = T5 - Q4$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_3.

S_0		S_1	
0_1	$H = 0_1$	$1_1 1$	$H = 11_2$
$0_0 1$	$H = 11_2$	$1_1 01$	$H = 101_3$
$01\bar{1}1$	$H = 11_2 \bar{1}1$	$1_1 00$	$H = 100_3$
$0_0 01$	$H = 101_3$		
$0_0 00$	$H = 100_3$		

Apply the method of exits to $T5$.

exit at X_3				
1_3				
101_3			101_3	
$0_0 01$			$1_1 01$	
$00_4 1$			$1:0$	
$1:0$	$0_1 01$	$0:L$		
	$1_4 01$			
	$0:R$	$1_2 101$		$10_3 1$
		$0_0 101$	$1_1 1101$	END
		$0:L$	$1:0$	100_3 required

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, $T5$ = no. 6 does not halt for standard starting configuration.

Hold out no. 7

$T4 = T5 - Q4$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_1.

S_0		S_1	
$0_0 01$	$H = 001_3$	$1_1 01$	$H = 101_3$
$0_0 00$	$H = 100_3$	$0_1 01$	$H = 001_3$
$0_0 1$	$H = 11_2$	$1_1 00$	$H = 100_3$
		$0_1 00$	$H = 000_3$
		$0_1 1$	$H = 01_2$
		$1_1 1$	$H = 11_2$

Apply the method of exits to $T5$.

exit at X_3

1_3

101_3		101_3	001_3	
$0_0 01$		$1_1 01$	$0_0 01$	
$00_4 1$		$1:0$	$00_4 1$	
$1:R$	$0:L$		$1:R$	$0:L$

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 7 does not halt for standard starting configuration.

Hold out no. 8

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_2 and S_4.

S_2		S_4	
$0\bar{1}10_2 0$	$H = 0_0\bar{1}100$	00_4	$H = 0_0 0$
$00_2 0$	$H = 0_0 00$	$0\bar{1}10_4$	$H = 0_0\bar{1}10$
$0_2 1$	$H = 01_3$	$0\bar{1}1_4 00$	$H = 0_0\bar{1}100$
1_2	$H = 1_2$	$1_4 01$	$H = 101_3$
		$1_4 1$	$H = 11_2$

Apply the method of exits to $T5$.

exit at X_3		
1_3		
01_3		101_3
$0_2 1$		$1_4 01$
$1_1 01$		$0_1 101$
$0_0 01$		$1_2 101$
$00_4 1$		$1_1 1101$
$0_1 01$		$0_0 1101$
$1_2 01$		END
$1_2 01$	$11_2 01$	$0_0\bar{1}100$, $0_0 00$, $0_0 0$, $0_0\bar{1}100$, $0_0\bar{1}10$ required
$1_1 101$	$1_1 101$	
$0_0 101$	$0_0 101$	
END	END	

$0_0\bar{1}100$, $0_0 00$, $0_0 0$, $0_0\bar{1}100$, $0_0\bar{1}10$ required

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 8 does not halt for standard starting configuration.

Hold out no. 9

$T4 = T5 - Q2$. To halt *T*5 must have

$$X_1 = 0_1$$
$$\to S_3 \to Q_2 \to \begin{Bmatrix} X_0 \\ X_1 \end{Bmatrix} \to S_3$$

So, we only need to consider S_3 in the complete criterion for *T*4.

S_3

$100_3 \quad H = 1_0 01$

$1_3 1 \quad H = 11_1$

$1_3 0 \quad H = 10_1$

$10\bar{1}0_3 \quad H = 1_0 0\bar{1}$

$000_3 \quad H = 1_1 01$

$00\bar{1}0_3 \quad H = 1_1 0\bar{1}$

Assuming exit at X_3 in *T*5, then by the third loop backwards we reach no further path backwards to S_3 .

The complete criterion for *T*5 = no. 9 is

$$\left.\begin{matrix} 1_1 01 \\ 0_2 1 \\ 1_3 \\ 11_2 01 \end{matrix}\right\} \overline{101}0$$

Standard starting configuration, $S_0 = \overline{\overline{0_0}}$, is not a halting configuration. Hence, no. 9 does not halt.

Hold out no. 10

$T4 = T5 - Q2$. Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criteria for S_1 and S_3.

S_1		S_3	
1_1	$H = 1_1$	11_3	$H = 1_4 1$
0_1	$H = 0_1$	$11\bar{0}0_3$	$H = 1_4 1\bar{0}0$
$0\bar{1}1_0$	$H = 1_1\bar{1}1$	$\bar{1}1101_3$	$H = 1_1\bar{1}101$
		$0\bar{1}1010\bar{0}0_3$	$H = 1_1\bar{1}1010\bar{0}0$

Apply the method of exits to *T*5.

exit at X_4							
1_4							
$1_4 1$	$1_4 1\bar{0}0$						
11_3	$11\bar{0}0_3$						
$0:R$	$11\bar{0}0_2 0$						
	$110_2 0$		$11\bar{0}00_2 0$				
	$11_1 00$		$11\bar{0}0_1 00$				
	$1:0$	END	$11\bar{0}1_2 00$				
		$1_1 1$	$111_2 00$			$11\bar{0}01_2 00$	
			$11_1 100$			$11\bar{0}0_1 100$	
			$011_0 00$	0110_4	$1:0$	$1101_2 100$	
			END	END		$110_1 1100$	$1:R$
						$111_2 1100$	
						$(1_1\bar{1}1)$	$(1_1\bar{1}10)$
						$11_1 11100$	$11_1 1110$
						$11111_0 00$	111110_4
						END	END

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, *T*5 = no. 10 does not halt for standard starting configuration.

Hold out no. 11

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_2.

S_2	
$001\bar{0}0_2 0$	$H = 0_0 01\bar{0}00$
$0\bar{1}101\bar{0}0_2 0$	$H = 0_1\bar{1}101\bar{0}00$
1_2	$H = 1_2$
$11\bar{0}0_2 0$	$H = 1_4 1\bar{0}00$

Apply the method of exits to $T5$.

exit at X_4		
1_4		
$1_4 1\bar{0}00$		
$1_1 11\bar{0}00$		
$1_2 111\bar{0}00$	$0_0 11\bar{0}00$	$0_0 1100$
$1_1 1111\bar{0}00$	END	END
LOOP	$0_0 0100$, $0_0 1101$ required	

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 11 does not halt for standard starting configuration.

Hold out no. 12

$T4 = T5 - Q4$. Apply the method of exits and write the complete criterion for *T*4. We are particularly interested in the complete criterion for S_1. We can then show that no. 12 is non-halting for standard starting configuration by two methods. (1) Input of $S_0 = \overline{\overline{0_0}}$ leads to an infinite loop. (2) We can write the complete criterion for *T*5 to obtain

$$\overline{101}\begin{cases} 11_2 \\ 1_1 1 \\ 11_4 \end{cases} \quad H = 1_0 01\overline{101}11$$

$$1\overline{01}1_1 00 \quad H = 1_0 01\overline{101}$$

Standard starting configuration, $S_0 = \overline{\overline{0_0}}$, is not a halting configuration; therefore, *T*5 = no. 12 does not halt for standard starting configuration.

Hold out no. 13

$T4 = T5 - Q1$. Method of exits and write the complete criterion for $T4$.

S_0		S_2	
0_0	$H = 0_0$	001_2	$H = 0_001$
1_01	$H = 01_3$	11_2	$H = 1_41$
S_3		S_4	
1_3	$H = 1_3$	1_4	$H = 1_4$
10_3	$H = 1_41$	00_4	$H = 0_00$
000_3	$H = 0_001$		

In this, however, it is only the criterion for S_2 that is relevant. Then, applying the method of exits to $T5$.

exit at X_1		
0_1		
1_30	1_40	$0:1$
NO PATH TO S_2	HALTS ON 1_41. NO PATH TO S_2.	

Hence, $T5$ = no. 13 does not halt.

Hold out no. 14

$T4 = T5 - Q0$. Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criterion for S_1.

S_1

$0\bar{1}0_1$	$H = 0_4 0$
$0\bar{1}1_1 00$	$H = 0_4 \bar{1}100$
$01_1 01$	$H = 0_4 100$
$1_1 1$	$H = 11_2$
00_1	$H = 0_4 0$

Apply the method of exits to *T*5.

exit at X_2		
1_2		
$1_1 1$		
$0_0 1$		
$0 : L$	$1_0 01$	$\overline{1_0}101$
	$10_4 1$	$0 : L$
	$101_1 00$	
	$100_0 00$	
	$10_4 000$	
	$100_1 00$	
	$0 : 1$	

Standard starting configuration, $S_0 = \overline{\overline{0_0}}$, is not a halting configuration of this tree. Hence, *T*5 = no. 14 does not halt for standard starting configuration.

Hold out no. 16

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_3 and S_4.

S_3		S_4	
1_3	$H = 1_3$	1_4	$H = 1_4$
10_3	$H = 1_1 0$	$0_4 1$	$H = 01_1$
000_3	$H = 1_1 00$	$0_4 0$	$H = 01_1$

Apply the method of exits to $T5$.

exit at X_3					
1_3					
$0_2 1$					
$1_1 01$					
$1_1 00$	$10_3 1$	$0_4 101$		$0_4 001$	
	$0:R$	$1_2 101$		$1_2 001$	
		$1_1 1101$		$1_1 1001$	
		$0_4 11101$	$0_4 01101$	$0_4 11001$	$0_4 01001$
		$0:1$	$0:1$	$0:1$	$0:1$

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, $T5$ = no. 16 does not halt for standard starting configuration.

Hold out no. 17

$T4 = T5 - Q4$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_1 and S_2.

S_1		S_2	
0_1	$H = 0_1$	10_2	$H = 1_3 0$
$1_1 0$	$H = 1_3 0$	$00_2 \overline{10}11$	$H = 01\overline{1}111_0$
$1_1 11$	$H = 111_0$	$1_2 0\overline{10}11$	$H = 01\overline{1}111_0$
$1_1 \overline{10}11$	$H = 1\overline{1}111_0$		

Apply the method of exits to $T5$.

exit at X_0

1_0							
111_0	$1\overline{1}111_0$	$01\overline{1}111_0$		$11\overline{1}111_0$			
$1_1 11$	$1_1 \overline{10}11$	$00_2 \overline{10}11$		$1_2 0\overline{10}11$			
$0 : L$	$0 : L$	$0_4 0\overline{10}11$		$10_4 \overline{10}11$			
		$1 : L$	$00_1 \overline{10}11$	$0 : L$	$101_3 0\overline{10}11$		
			$0 : L$		$101_1 0\overline{10}11$		$1010_2 \overline{10}11$
					$1010_4 \overline{10}11$		$0 : R$
					$0 : L$	$10\overline{10}1_3 1$	
						END	
						$1_3 0$ required	

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 17 does not halt for standard starting configuration.

Hold out no. 18

$T4 = T5 - Q1$. (The choice of $Q1$ is to ensure maximum "disruption" to the original machine, while leaving it connected.) Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criterion for S_0. Apply the method of exits to *T*5. *T*5 closes almost immediately.

exit at X_4	complete criterion halting configurations for $T5$	
0_4	$0\overline{1_0}$	01_0
01_0	$01\overline{1_4}$	011_4
010_1	$01\bar{1}0_1$	
011_4		
0111_0		
01110_1		
01111_4		
LOOP		
$1_4 \to 11_0 \to 111_4$		

The tree is finite (closes) and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$. Hence, *T*5 = no. 18 does not halt for standard starting configuration.

Hold out no. 22

$T4 = T5 - Q3$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_1.

S_0		S_1	
1_0	$H = 1_0$	0_1	$H = 1_0$
$0_0 0$	$H = 10_2$	$1_1 0$	$H = 10_2$
$0_0 100$	$H = 1100_2$	$1_1 100$	$H = 1100_2$
$0_0 10\overline{10}0$	$H = 110\bar{1}00_2$	$0_1 00$	$H = 100_2$
$0_0 1\overline{01}1$	$H = 11\overline{01}1_4$	$0_1 0\overline{10}0$	$H = 10\overline{10}0_2$
		$1_1 10\overline{10}0$	$H = 110\overline{10}0_2$
		$1_1 \overline{10}0$	$H = 1\overline{10}0_2$
		$1_1 1\overline{01}1$	$H = 11\overline{01}1_4$
		$0_1 \overline{01}1$	$H = 1\overline{01}1_4$

Apply the method of exits to $T5$.

exit at X_4

1_4			
$11\overline{01}1_4$	$11\overline{01}1_4$	$1\overline{01}1_4$	
$0_0 1\overline{01}1$	$1_1 1\overline{01}1$	$0_1 \overline{01}1$	
$1_3 1\overline{01}1$	$0 : L$	$00_3 1\overline{01}1$	
$0 : L$		$01_0 1\overline{01}1$	
$1 : 0$		$00_1 1\overline{01}1$	$01_0 1\overline{01}1$
		$01_3 1\overline{01}1$	$1 : 0$
		$0 : L$ $1 : 0$	

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, $T5$ = no. 22 does not halt for standard starting configuration.

Hold out no. 23

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_2 and S_4.

S_2		S_4	
00_2	$H = 0_3 0$	1_0	$H = 1_4$
$0\overline{11}0_2$	$H = 0_3\overline{11}0$	010_4	$H = 0_3 10$
$0\overline{11}1_2 0$	$H = 0_3\overline{11}10$	$0\overline{11}10_4$	$H = 0_3\overline{11}10$
010_2	$H = 0_0 10$	00_4	$H = 0_0 0$
$0\overline{11}10_2$	$H = 0_0\overline{11}10_2$	$0\overline{11}0_4$	$H = 0_0\overline{11}0$

Apply the method of exits to $T5$.

exit at X_3

0_3

$0_3 1$	$0_3\overline{11}1$	$0_3 0$	$0_3\overline{11}0$	$0_3\overline{11}10$	$0_3 10$	$0_3\overline{11}10$
01_0	$0\overline{11}1_0$	00_2	$0\overline{11}0_2$	$0\overline{11}1_2 0$	010_4	$0\overline{11}10_4$
END	END	$1 : R$	$0\overline{11}11_1 0$	$0\overline{11}11_1 10$	$1 : R$	$0 : R$
			$0\overline{11}10_0 0$	$0\overline{11}10_0 10$		
			$0\overline{11}100_4$	$0\overline{11}1010_2$		
			$0\overline{11}10_1 0$	$0\overline{11}101_1 0$		
			$0\overline{11}11_4 0$	$0\overline{11}100_0 0$		
			$0 : R$	$0\overline{11}1000_4$		
				$0\overline{11}100_1 0$		
				$0\overline{11}101_4 0$		
				$0\overline{11}10_1 10$		
				$0\overline{11}11_4 10$		
				$0 : R$		

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, $T5$ = no. 23 does not halt for standard starting configuration.

Hold out no. 24

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion.

S_1		S_3	
100_1	$H = 1_0 11$	11_3	$H = 1_2 1$
$10\bar{1}0_1$	$H = 1_0 1\bar{1}$	01_3	$H = 1_1 1$
1_1	$H = 1_1$		
000_1	$H = 1_1 11$		
$00\bar{1}0_1$	$H = 1_1 11\bar{1}$		

Apply the method of exits to $T5$. It is immediate that there is no backward path from the exit at $X_3 = 0_3$ to any input that would lead to a feedback loop. The tree is closes immediately and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$. Hence, $T5$ = no. 24 does not halt for standard starting configuration.

Hold out no. 25

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion.

S_1		S_3	
1_1	$H = 1_1$	11_3	$H = 1_0 1$
00_1	$H = 01_1$	01_3	$H = 1_1 1$
$0\bar{1}0_1$	$H = 01_1\bar{1}$		

Apply the method of exits to $T5$. It is immediate that there is no backward path from the exit at $X_3 = 0_3$ to any input that would lead to a feedback loop. The tree is closes immediately and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$. Hence, $T5$ = no. 24 does not halt for standard starting configuration.

Hold out no. 26

$T4 = T5 - Q3$. Apply the method of exits to *T*4 and write its complete criterion. This program has a sub-routine. We are particularly interested in the criteri0n for S_4.

S_4

$1_4 \qquad H = 1_4$

$0_4\left\{\begin{matrix}\overline{100}\\ \overline{11}\end{matrix}\right\}0 \qquad H = 0\left\{\begin{matrix}\overline{101}\\ \overline{11}\end{matrix}\right\}0_2 \qquad 0_40 \rightarrow 00_2$

$0_4\left\{\begin{matrix}\overline{100}\\ \overline{11}\end{matrix}\right\}101 \qquad H = 0\left\{\begin{matrix}\overline{101}\\ \overline{11}\end{matrix}\right\}101_0 \qquad 0_411101 \rightarrow 011101_0$

Apply the method of exits to *T*5, which exits at $X_0 = 1_0$. The halting configurations are

$$\left.\begin{matrix}1_1\\ 0_4\\ 0_10\\ 1_200\\ 1_21\end{matrix}\right\}\left\{\begin{matrix}\overline{100}\\ \overline{11}\end{matrix}\right\}101 \qquad H = \left.\begin{matrix}1\\ 0\\ 00\\ 100\\ 11\end{matrix}\right\}\left\{\begin{matrix}\overline{101}\\ \overline{11}\end{matrix}\right\}101_0$$

The tree has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$, which is not among the halting configurations. Hence, *T*5 = no. 26 does not halt for standard starting configuration.

Hold out no. 27

$T4 = T5 - Q3$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_1 and S_4.

S_1		S_4	
0_1	$H = 0_1$	1_4	$H = 1_4$
$1_1 0$	$H = 10_2$	0_4	$H = 0_4$
$1_1 100$	$H = 1110_2$		
$1_1 \overline{10} 100$	$H = 1_1 \overline{11} 110_2$		

Apply the method of exits to $T5$.

exit at X_1

0_1

$0_3 0$

$1 : L$	$0 : 1$	00_4		
		001_3		
		0011_4	0010_4	000_2
		00111_3	00101_3	END
		LOOP	LOOP	

The halting configurations are

$$0 \begin{Bmatrix} \overline{11} \\ \overline{10} \end{Bmatrix} \begin{Bmatrix} 1_3 \\ 11_4 \\ 10_4 \\ 0_2 \end{Bmatrix}$$

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 27 does not halt for standard starting configuration.

Hold out no. 28

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_3.

S_0		S_1	
0_0	$H = 1_1$	$0_3 1$	$H = 01_1$
$0\overline{11}1_0$	$H = 01_1 10$	011_3	$H = 01_1 1$
$0\overline{11}11_0$	$H = 0_4\overline{11}10$	$0\overline{11}11_3$	$H = 01_1 1\overline{11}$
		$0\overline{11}1_3$	$H = 0_4\overline{11}1$

Apply the method of exits to $T5$.

exit at X_4	
0_4	
$0_4\overline{11}10$	$0_4\overline{11}1$
$0\overline{11}10_1$	$0\overline{11}1_3$
$0\overline{11}1_2$	$0 : R$
$\overline{11}11_1 10$	
END	
01_1 required	

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, $T5$ = no. 28 does not halt for standard starting configuration.

Hold out no. 29

$T4 = T5 - Q1$. Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criteria for S_2 and S_3.

S_2		S_3	
$1_2 0$	$H = 10_0$	$0_3 1$	$H = 01_1$
$1_2 1$	$H = 11_0$	$0_3 0$	$H = 00_0$
00_2	$H = 0_4 1$	01_3	$H = 0_4 1$
$0\overline{11}0_2$	$H = 0_4 \overline{11}1$	$0\overline{11}1_3$	$H = 0_4 \overline{11}1$

Apply the method of exits to *T*5.

exit at X_4

0_4			
$0_4 1$	$0_4 \overline{11}1$	$0_4 1$	$0_4 \overline{11}1$
00_2	$0\overline{11}0_2$	01_3	$0\overline{11}1_3$
$1:R$	$0\overline{11_1}0$	010_1	$0\overline{11}10_1$
	$0\overline{10_0}0$	011_0	$0\overline{11}11_0$
	$0\overline{1_2 0}0$	$01_2 1$	$0\overline{11}1_2 1$
	$1:R$	$1:R$	$0\overline{10_0 1}1$
			$0\overline{1_2 01}1$
			$1:R$

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, *T*5 = no. 29 does not halt for standard starting configuration.

Hold out no. 30

$T4 = T5 - Q3$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_4.

S_0		S_4	
$0_0 10$	$H = 100_1$	$0_4 0$	$H = 00_1$
$\overline{1_0} 010$	$H = \bar{1}100_1$	$0_4 10$	$H = 110_2$
$0_0 0$	$H = 10_2$	$0_4 \overline{11} 0$	$H = 1\overline{11}0_1$
$0_0 110$	$H = 1010_2$		
$0_0 \overline{11} 0$	$H = 1\overline{01}0_2$		

Apply the method of exits to $T5$.

0_1						
S_0						S_4
100_1	$\bar{1}100_1$					00_1
$0_0 10$	$\overline{1_0}\, 010$					$0_4 0$
$0:1$	$1_0 \bar{1} 010$		$1_0 010$			$1:L$
	$0_3 \bar{1} 010$		END			
	$0:1$	$0_3 101$		$0_3 1\bar{1}01$		
		$0:1$	$01_4 01$	$01_4 \bar{1} 01$	$0:1$	
			END	END		

The tree is closes and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$. Hence, $T5$ = no. 30 does not halt for standard starting configuration.

Hold out no. 31

$T4 = T5 - Q4$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0. Apply the method of exits to $T5$.

exit at X_2

X_2 1_2

X_2 011_2 required to exit

S_0 $0_0 0 \left\{ \begin{matrix} \overline{\bar{1}010} \\ \overline{010} \end{matrix} \right\} 011$

$00_4 \left\{ \begin{matrix} \overline{\bar{1}010} \\ \overline{010} \end{matrix} \right\} 011$

X_0 $001_0 \bar{1}010 \left\{ \begin{matrix} \overline{\bar{1}010} \\ \overline{010} \end{matrix} \right\} 011$ or X_1 $000_1 11 \left\{ \begin{matrix} \overline{\bar{1}010} \\ \overline{010} \end{matrix} \right\} 011$ $000_1 11$

END

TAPE CONTRADICTION must have 100_1...

TAPE CONTRADICTION must have 100_1...

The tree is closes and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$. Hence, $T5$ = no. 31 does not halt for standard starting configuration.

Hold out no. 32

$T4 = T5 - Q0$. Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criterion for S_1. The tree generated includes repeat loops. The halting configurations for S_1 are

$$\left.\begin{matrix} 1_1 0\bar{1}0 \\ 1_1 0 \end{matrix}\right\} \left\{\begin{matrix} \overline{10\bar{1}0} \\ \overline{10} \end{matrix}\right\} 0 \qquad H = \bar{1}0_4 0$$

$$\left.\begin{matrix} 1_1 0\bar{1}0 \\ 1_1 0 \end{matrix}\right\} \left\{\begin{matrix} \overline{10\bar{1}0} \\ \overline{10} \end{matrix}\right\} 11 \qquad H = \bar{1}011_2$$

We note in particular

$$\begin{cases} X_4 = 0_4 0 \\ S_1 = 00_1 \end{cases} \qquad \begin{cases} X_2 = 11_2 \\ S_1 = 1_1 1 \end{cases}$$

When we apply the method of exits to *T*5, the tree always has a terminal 11. We reach

$1_0 11$ or $0_4 11$

$0 : L$ $\quad$ $0 : L$

Hence, standard starting configuration, $S_0 = \overline{\overline{0_0}}$, is not a halting configuration of this tree. Hence, *T*5 = no. 32 does not halt for standard starting configuration.

Hold out no. 33

$T4 = T5 - Q3$. Apply the method of exits to $T4$ and write its complete criterion. There is a sub-routine.

SUB ROUTINE

$\overline{11_4}$

$\overline{11}1_0$

$\overline{11}10_1$

$0:1 \qquad \overline{11}11_4 \qquad \overline{11}1_2 0 \qquad \overline{11_4} \to \begin{cases} \overline{11_4} \\ 1_2\overline{11}0 \\ \overline{11_1}10 \\ \overline{11}1_2 0 \\ \overline{11}1_2 1\overline{11}10 \\ \overline{11}11_1\overline{11}10 \end{cases}$

$\overline{11_1}10$

$\overline{1_2 1}10$

$\overline{11}10$

We are particularly interested in the criteria for S_1. Apply the method of exits to $T5$.

exit at X_4

0_4

010_1	01110_1	$0\overline{11}11_1 10$	$0\overline{11}1_1\overline{11}10$	$011_1\overline{11}10$
$0:R$	$0:R$	$0:R$	$0:R$	$0:R$

In order to halt we must have a path

$$S_0 = \overline{\overline{0_0}}$$

$$\left\{\overline{X_2 \to Q_3 \to S_1}\right\} \to X_2 \to Q_3 \to S_1$$

So, we must be able to trace back from X_4 to S_1. But the above tree shows that there is no path from Q_3 to any possible candidate for S_1, where we are lead immediately to a tape contradiction $0:R$. The tree is closes and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$. Hence, $T5$ = no. 33 does not halt for standard starting configuration.

Hold out no. 34

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_2 and S_4.

S_2		S_4	
1_2	$H = 1_2$	01_4	$H = 0_0 1$
0_2	$H = 1_3$	$0\overline{01}0_4$	$H = 0_0\overline{01}0$
		$0\overline{01}01_0$	$H = 0_0\overline{01}01$
		1_4	$H = 0_3$
		$1\overline{10}1_4$	$H = 0_3\overline{10}10$

Apply the method of exits to $T5$.

exit at X_2

1_2						
$1_1 1$						
$0_3 11$	$1_3 11$			$0_0 1$		
$1_4 11$	$0_2 11$			00_4		
$0 : L$	$1_1 011$			000_1		
	$1_3 1011$	$0_3 1011$	$0_0 011$	$00_3 0$	$0:1$	$1:R$
	LOOP	$1_4 1011$	END	$01_4 0$		
		$0 : L$	$0_0 1$ required	010_1		
				$01_3 0$	$0:1$	$1:R$
				$00_2 0$		
				$1 : R$		

The halting loop is

$$\left.\begin{matrix} 1_3 \\ 0_3 \\ 1_4 \\ 0_2 \\ 1_1 0 \end{matrix}\right\} \overline{1}011$$

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 34 does not halt for standard starting configuration.

Hold out no. 35

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_2 and S_3.

S_2		S_3	
0_2	$H = 1_3$	000_3	$H = 0_0 00$
1_2	$H = 1_2$	$00\overline{10}0_3$	$H = 0_0 0\overline{10}0$
		10_3	$H = 1_4 0$
		1100_3	$H = 1_4 100$
		$1\overline{10}0_3$	$H = 1_4\overline{10}0$

Apply the method of exits to $T5$.

exit at X_2					
X_2	1_2				
S_2	1_2	here $S_2 = X_2$			
	$1_1 1$				
	$1:0$	$0_3 11$			$0_0 1$
		$0_1 011$			TAPE CONTRADICTION
		$0:1$	$1_4 011$	$0_3 0011$	$0_0 0\overline{10}0$ required
			$10_3 11$	$0_1 00011$	
			$0:R$	$0_1 0\overline{00}11$	
				$0_3\overline{00}11$	
				LOOP	

Halting configurations

1_2 $1_1 1$ $0_3 11$ $0_1 0\overline{00}11$ $0_3\overline{00}11$ $1_4 011$ $10_3 11$ $0_0 1$

The tree is closes and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$, which is not among the halting configurations. Hence, $T5$ = no. 35 does not halt for standard starting configuration.

Hold out no. 36

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_2 and S_3.

S_2		S_3	
1_2	$H = 1_2$	$0_3 1$	$H = 11_4$

Apply the method of exits to $T5$.

exit at X_2

1_2						
$1_1 1$						
$1:0$	$1_3 11$	$0_0 1-$				
	$0_1 111$	END - no path to S_2, S_3				
	$0:1$	$1_4 111$		$1_3 0111$		
		$0_3 1111$		$0_1 10111$		
		$0_1 11111$		$0:1$	$1_4 10111$	$1_3 010111$
		$1_4 11111$	$1_3 011111$		LOOP	LOOP
		LOOP	LOOP			

Halting configurations

$$\left.\begin{matrix} 1_4 \\ 0_1 \\ 1_3 0 \end{matrix}\right\} \left\{\begin{matrix} \overline{11} \\ \overline{01} \end{matrix}\right\} 1$$

The tree is closes and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$, which is not among the halting configurations. Hence, $T5$ = no. 36 does not halt for standard starting configuration.

Hold out no. 37

$T4 = T5 - Q3$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_4.

S_4

$1_4 \quad H = 1_4$

$0\overline{1}110_4 \quad H = 01\overline{0}00_1$

$010_4 \quad H = 100_1$

Apply the method of exits to $T5$.

exit at X_4

1_4

11_3

$0:R \quad 11\overline{0}0_3$

$11\overline{0}0_10$

$1100_10 \quad \overline{0} = 0$ required

1010_40

$1:L$

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, $T5$ = no. 37 does not halt for standard starting configuration.

Hold out no. 38

$T4 = T5 - Q2$. Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criteria for S_0 and S_1.

S_0		S_1	
1_0	$H = 1_0$	1_1	$H = 1_1$
		10_1	$H = 1_3 0$
		000_1	$H = 0_4 00$

Apply the method of exits to *T*5.

exit at X_0

X_0	1_0					
S_0	1_0	$S_0 = X_0$				
	$1_2 1$					
	$1:0$	$0_4 1$	$1_1 11$			
		END	$0_2 111$			
		$0_4 00$	$0_2 111$			
		required	$0:1$	$1_3 111$	$1_1 0111$	
				END	$0_2 10111$	
				$1_3 0$	$1_3 10111$	$1_1 010111$
				required	LOOP	LOOP

Halting configurations

1_0 $1_2 1$ $0_4 1$ $1_1 11$ $0_2 111$ $1_3 111$

$\left.\begin{matrix} 0_2 1 \\ 1_1 \\ 1_3 1 \end{matrix}\right\} \overline{01}\,11$

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$, which is not among the halting configurations. Hence, *T*5 = no. 38 does not halt for standard starting configuration.

Hold out no. 39

$T4 = T5 - Q1$. Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criteria for S_2 and S_3.

S_2		S_3	
$1_2 1$	$H = 11_0$	00_3	$H = 00_0$
$1_2 0$	$H = 10_0$	0010_3	$H = 00_0 10$
1_2	$H = 1_2$	$0\overline{01}0_3$	$H = 00_0\overline{01}0$

Apply the method of exits to *T*5.

exit at X_0

1_0		
$1_2 1$	S_3 no path	
$1_1 11$		
$1_2 111$	$0_0 11$	
$1_1 1111$	$00_3 11$	0010_3
$1_1\overline{11}$	$0 : L$	END
$1_2 1\overline{11}$		
LOOP		

Halting configurations

1_0 $1_2 1$ $1_1 11$ $1_1\overline{11}$ $1_2 1\overline{11}$ $0_0 11$ $00_3 11$

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$, which is not among the halting configurations. Hence, *T*5 = no. 38 does not halt for standard starting configuration.

Hold out no. 40

$T4 = T5 - Q3$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_4.

S_4

10_4	$H = 1_0 0$
$1_4 01$	$H = 101_2$
01_4	$H = 0_1 1$
$1_4 100$	$H = 1100_1$

Apply the method of exits to $T5$.

exit at X_2

1_2

101_2 required (RQ)

$1_4 01$

$0_3 01$

$00_1 1$ $1:L$

001_4

000_3

0000_1	0001_0	
$0000_1 1$ RQ	$0001_0 0$ RQ	
00001_4	00010_4	
LOOP	00011_3	
	000110_1	000111_0
	LOOP	LOOP

The halting loop is

$$00\begin{Bmatrix}\overline{00}\\ \overline{11}\end{Bmatrix}\begin{cases}1_4\\ 0_3\\ 00_1 1\\ 01_0 0\\ 010_4\\ 011_3\end{cases}$$

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, $T5$ = no. 40 does not halt for standard starting configuration.

Hold out no. 41

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_1 and S_4.

S_1		S_4	
1_1	$H = 1_1$	1_4	$H = 1_4$
10_1	$H = 1_3 0$	10_4	$H = 1_0 0$

Apply the method of exits to $T5$.

exit at X_0			
1_0			
10_4			
101_2			
1011_4	$1:R$		100_3
10111_2			END $1_3 1$ required
101111_4	10110_3	$1011_1 1$	
LOOP	END	$0:R$	

The tree has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$, which is not among the halting configurations. Hence, $T5$ = no. 41 does not halt for standard starting configuration.

Hold out no. 42

$T4 = T5 - Q3$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_1.

S_0		S_1	
0_0	$H = 1_1$	1_1	$H = 1_1$
$1_0 1$	$H = 11_4$	$0_1 1\overline{01}1$	$H = 01\overline{01}1_1$
$1_0 0$	$H = 10_4$	$1_2\overline{01}1$	$H = 1\overline{01}1_1$

Apply the method of exits to $T5$.

exit at X_2

1_2

$1_2\overline{01}1$ required

$1\overline{01}1_1$

$1\overline{01}01_3 1$

$1\overline{01}00_4 1$ $0:L$ $1:0$

END

11_4 or 10_4 required

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 42 does not halt for standard starting configuration.

Hold out no. 43

$T4 = T5 - Q3$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_0.

S_0

$01_01\begin{Bmatrix}\overline{10}\\ \overline{11}\end{Bmatrix}0 \qquad H = 111\begin{Bmatrix}\overline{10}\\ \overline{11}\end{Bmatrix}0_1$

$\begin{matrix}0_01\\ 0_00\end{matrix}\Bigg\}\begin{Bmatrix}\overline{10}\\ \overline{11}\end{Bmatrix}0 \qquad H = \begin{matrix}11\\ 10\end{matrix}\Bigg\}\begin{Bmatrix}\overline{10}\\ \overline{11}\end{Bmatrix}0_1$

$11_0 \qquad H = 1_41$

Apply the method of exits to $T5$.

exit at X_3

1_3

$10_1 \qquad 1:0$

$111\overline{11}0_1$ required

$01_01\overline{11}0$

$0:L$

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 43 does not halt for standard starting configuration.

Hold out no. 44

$T4 = T5 - Q3$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_4.

S_0		S_4	
$0_0 1$	$H = 11_2$	1_4	$H = 1_4$
$1_0 11$	$H = 111_2$	$0_4 11$	$H = 111_2$
$0_0 0\overline{10}0$	$H = 10\overline{10}0_1$	$0_4 \overline{10}0$	$H = 1\overline{10}0_1$

Apply the method of exits to $T5$.

exit at X_2

1_2				
11_2	111_2	111_2		
$0_0 1$	$1_0 11$	$0_4 11$		
$0 : L$	$0 : L$	$01_3 1$		
		$00_4 1$	$0 : L$	
		000_4		0010_1
		0001_3		END 100_1 required
		0000_4	00010_1	
		00001_3		
		LOOP		

The halting loop is

$$0\overline{0}\begin{cases}1_3\\0_4\\10_1\end{cases}$$

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 44 does not halt for standard starting configuration.

Hold out no. 45

$T4 = T5 - Q4$. Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criteria for S_0 and S_3.

S_0		S_3	
1_0	$H = 1_0$	1_3	$H = 1_3$
$0_0 00$	$H = 11_0$	0_3	$H = 1_0$
$0_0\overline{01}00$	$H = 1\overline{01}1_0 0$		
$0_0 1$	$H = 11_2$		
$0_0 011$	$H = 1011_2$		
$0_0\overline{01}1$	$H = 1\overline{01}1_2$		

Apply the method of exits to *T*5. The tree is a little longer and more involved that many others, but it does close and is finite.

Halting configurations

1_3		
11_4		
111_0		
1110_4		
1111_2		
$110_0 1$		
11101_0		
111010_4		
111011_2		1110101_0
$110_0 011$		$111\overline{01}0_4$
$110_0\overline{01}1$	$111\overline{01}1_2$	$111\overline{01}01_0$
$1100_4 11$		$111\overline{01}11_2$
$1101_2 11$		
$11001_0 1$		
$10_0 0111$		
$1100_0 11$		
$1011_2 11$		
111011_2		
$11011_2 1$		

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$, which is not among the halting configurations. Hence, *T*5 = no. 45 does not halt for standard starting configuration.

Hold out no. 46

$T4 = T5 - Q2$. Apply the method of exits to *T*4 and write its complete criterion. *T*4 has exits at X_0, X_1, X_3 and X_4. Apply the method of exits to *T*5. The tree is a little longer and more involved that many others, but it does close and is finite.

Halting configurations

$$1_0 \quad 1_2 1 \quad 0_3 1 \quad 0_2 01 \quad 1_4 01 \quad 1_1 11 \quad 0_0 11 \quad 1_2 011$$

$$\left.\begin{array}{l} 1_1 0 \\ 0_2 0 \\ 0_3 \\ 1_2 0 \\ 0_0 0 \\ 1_4 \end{array}\right\} \overline{00}\,01 \qquad \left.\begin{array}{l} 1_1 0 \\ 0_2 0 \\ 0_3 \\ 1_2 0 \\ 0_0 0 \\ 1_4 \end{array}\right\} \left\{\overline{00}\right\}\left\{\overline{10}\right\}11$$

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$, which is not among the halting configurations. Hence, *T*5 = no. 46 does not halt for standard starting configuration.

Hold out no. 47

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particular interested in the criteria for S_0 and S_1.

S_0		S_1	
1_0	$H = 1_0$	1_1	$H = 1_1$
0_0	$H = 1_1$	10_1	$H = 1_4 1$
		1100_1	$H = 1_4 101$
		$11\overline{01}00_1$	$H = 1_4 1\overline{01}01$

Apply the method of exits to $T5$.

Halting configurations for $T5$

1_0	
$1_2 1$	
$1_1 11$	
$0_2 111$	$0_2 1\overline{01}11$
$1_4 111$	$1_4 1\overline{01}11$
$10_1 11$	$1_1\overline{01}11$
$1_1 011$	

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$, which is not among the halting configurations. Hence, $T5$ = no. 47 does not halt for standard starting configuration.

Hold out no. 48

$T4 = T5 - Q3$. Apply the method of exits to $T4$ and write its complete criterion. We are particular interested in the criteria for S_0 and S_4.

S_0		S_4	
$0_0 1$	$H = 11_2$	1_4	$H = 1_4$
$0_0 011$	$H = 1011_2$		
$0_0 \overline{01}1$	$H = 1\overline{01}1_2$		
$0_0 00$	$H = 100_1$		
$0_0 0100$	$H = 10100_1$		
$0_0 \overline{01}00$	$H = 1\overline{01}00_1$		

Apply the method of exits to $T5$.

Halting configurations for $T5$

1_4	
$1_3 1$	
$0_1 1$	
11_0	
110_3	
111_2	
1101_0	
11010_3	$11\overline{01}0_3$
110101_0	$11\overline{01}01_0$
11011_2	$11\overline{01}1_2$

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$, which is not among the halting configurations. Hence, $T5$ = no. 47 does not halt for standard starting configuration.

Hold out no. 50

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_2.

S_2

1_2	$H = 1_2$
$0\overline{001}0_2$	$H = 0_0\overline{101}0$
$101\overline{001}0_2$	$H = 1_4 01\overline{101}0$
00_2	$H = 0_0 0$
1010_2	$H = 1_4 010$

Apply the method of exits to $T5$.

exit at X_4

1_4

$1_4 01\overline{101}0$ required

$101\overline{001}0_2$

$101\overline{001}001_1 0$	$0 : R$
$101\overline{001}000_0 0$	$1 : R$
$101\overline{001}0000_2$	
$101\overline{001}000_1 0$	$1 : R$

$1 : R$

$0 : 1$

The tree has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$, which is not among the halting configurations. Hence, $T5$ = no. 50 does not halt for standard starting configuration.

Hold out no. 51

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_1.

S_0		S_1	
0_0	$H = 0_0$	1_1	$H = 1_1$
$0\overline{1}111_0$	$H = 1_1\overline{1}111$	0_1	$H = 0_1$
101_0	$H = 1_4 01$		
001_0	$H = 0_4 01$		
$00\overline{1}11_0$	$H = 0_4 0\overline{1}11$		
$10\overline{1}11_0$	$H = 1_4 0\overline{1}11$		

Apply the method of exits to $T5$.

exit at X_4

1_4

$1_4 0\overline{1}11$ required

$10\overline{1}11_0$

$10\overline{1}110_2$

$10\overline{1}11_1 0$ $0:R$ $0:1$

$10\overline{1}111_2 10$

$101_1 1\overline{1}110$

$101\overline{1}111_0 0$

$101\overline{1}1110_2$ $0:R$ $0:1$

$101\overline{1}111_1 0$

$101_1 1\overline{1}110$

LOOP

$101_1 \overline{1}10$

The tree has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$, which is not among the halting configurations. Hence, $T5$ = no. 51 does not halt for standard starting configuration.

Hold out no. 52

$T4 = T5 - Q1$. Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criterion for S_2.

S_2
$1_2 \quad H = 1_2$
$0\overline{11}0_2 \quad H = 0_0\overline{11}0 \qquad 00_2 \to 0_0 0$
$00\overline{11}0_2 \quad H = 0_4 0\overline{11}0$
$10\overline{11}0_2 \quad H = 1_4 0\overline{11}0 \qquad 100_2 \to 1_4 00$

If we apply the method of exits to *T*5 for the exit at X_4 the resultant tree has many branches. However, the tree closes and is finite.

To show that *T*5 is non-halting, we consider the input of standard starting configuration.

$S_0 \quad \overline{\overline{0_0}}$
$X_0 \quad \overline{\overline{0_0}}$
$\overset{=}{0}1_1\overset{=}{0}$
$\overset{=}{0}10_2\overset{=}{0}$

This last is not among the halting configurations for *T*4. To exit *T*4 we require $\overline{11}0_2$. Hence, *T*5 = no. 52 does not halt for standard starting configuration.

Hold out no. 53

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_2.

S_2
00_2 $H = 0_0 0$
1_2 $H = 0_4$

Apply the method of exits to $T5$.

exit at X_1
0_1
$0_4 0$ $1_4 0$ $0:1$
$1_2 0$ END
$1_1 10$
$0_4 110$ $1_4 110$ $0_0 10$
$1_2 110$ END END $0_0 0$ required
$1_2 110$
LOOP

The halting loop is

$$\left.\begin{matrix} 1_2 \\ 0_4 \\ 1_1 1 \end{matrix}\right\} \overline{1}10$$

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 53 does not halt for standard starting configuration.

Hold out no. 62

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_1.

S_0		S_1	
$\overline{1_0}\,0$	$H = \bar{1}1_1$	$0_1\bar{1}01$	$H = 0\bar{1}01_4$
0_0	$H = 1_1$	0_101	$H = 001_4$
		$0_1\bar{1}00$	$H = 0\bar{1}00_4$
		0_100	$H = 000_4$

Apply the method of exits to $T5$. The halting configurations for $T5$ are

1_4	
0_101	$0_1\bar{1}01$
1_201	$1_2\bar{1}01$
1_1101	$1_1\bar{1}01$
10_41	

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$, which is not among the halting configurations. Hence, $T5$ = no. 47 does not halt for standard starting configuration.

Hold out no. 64

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_2 and S_4.

S_2

$0\bar{1}0_2 \quad H = 0_0\bar{1}0$

$00_2 \quad H = 0_00$

$1_20 \quad H = 10_3$

S_4

$$\left.\begin{array}{l} 0\bar{1}10_4 \\ 01_4\bar{1}0 \\ 0\bar{1}1_4\bar{1}0 \\ 0\bar{1}1_40 \end{array}\right\} H = 0_0\bar{1}0$$

$$\left.\begin{array}{l} 010_4 \\ 01_40 \end{array}\right\} H = 0_010$$

Apply the method of exits to $T5$.

exit at X_3

0_3

1_20

1_110

0_010

010_4

$0 : R$

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$, which is not among the halting configurations. Hence, $T5$ = no. 64 does not halt for standard starting configuration.

Hold out no. 65

$T4 = T5 - Q3$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_4.

S_4
1_4 $\quad H = 1_4$
$0_4\overline{0\bar{1}01}0\bar{1}00$ $\quad H = 1\overline{1\bar{1}01}1\bar{1}00_2$
$0_4\overline{0\bar{1}01}1$ $\quad H = 1\overline{1\bar{1}01}1_0$

Apply the method of exits to $T5$.

exit at X_0
1_0
$0_4\overline{0\bar{1}01}1$ $\quad 0_41$
$0\overline{0_3\bar{1}01}1$ $\quad 0:L$
$0\overline{01_4\bar{1}01}1$ $\quad 0:L$
$0:L$

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$, which is not among the halting configurations. Hence, $T5$ = no. 66 does not halt for standard starting configuration.

Hold out no. 66

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_3 and S_4.

S_3		S_4	
1_3	$H = 1_3$	11_4	$H = 1_0 1$
		$01_4\bar{1}0$	$H = 11\bar{1}0_1$
		$0_4\bar{0}1\bar{1}0$	$H = \bar{0}11\bar{1}0_1$

Apply the method of exits to $T5$. $T5$ closes almost immediately.

exit at X_3
1_3
10_2
101_0 $\quad 0 : R$
$101_0 1$ required
1011_4
10111_2
101111_0 $\quad 0 : R$
LOOP

The halting configurations are

$$10\overline{111}\begin{cases} 1_0 1 \\ 11_4 \\ 111_2 \end{cases}$$

The standard starting configuration, $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 66 does not halt for standard starting configuration.

Hold out no. 67

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_3.

S_0		S_3	
$0_0 0$	$H = 10_1$	1_3	$H = 1_3$
$0_0 \bar{1} 0$	$H = 1\bar{1}0_1$	010_3	$H = 110_1$
1_0	$H = 1_0$	$00_3\bar{0}1\bar{1}0$	$H = 00\bar{0}11\bar{1}0_1$
		$00_3\bar{0}10$	$H = \bar{0}00110_1$

Apply the method of exits to $T5$.

exit at X_3

1_3

11_2 $\quad 0:R$

111_0

1111_2 $\quad 0:R$

11111_0

The halting configurations are

$\overline{1_3} \qquad \overline{1_2} \qquad \overline{1_0}$

The standard starting configuration, $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 66 does not halt for standard starting configuration.

Hold out no. 70

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_2.

S_2	
$01_2 0$	$H = 010_0$
$011_2 \overline{11}10$	$H = 01010_0$
0_2	$H = 0_2$
011_2	$H = 0_2 11$
$0_2 \overline{11}11_2$	$H = 0\overline{11}11_2$

Apply the method of exits to $T5$. It is immediate that there is no backwards path from the exit $X_4 = 0_4$ to S_2. The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$, which is not among the halting configurations. Hence, $T5$ = no. 70 does not halt for standard starting configuration.

Hold out no. 72

$T4 = T5 - Q2$. Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criteria for S_3 and S_4.

S_3		S_4	
$01_3\bar{1}0$	$H = 11\bar{1}0_1$	1_4	$H = 1_4$
01_30	$H = 110_1$	110_4	$H = 1_010$
$0_3\bar{0}1\bar{1}0$	$H = \bar{0}11\bar{1}0_1$		
$0_3\bar{0}10$	$H = \bar{0}110_1$		

Apply the method of exits to *T*5.

exit at X_4

1_4	
10_2	
100_1	100_0
END	END
110_1 required	no path to S_3 or S_4

The halting configurations are

1_4 10_2 100_1 100_0

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, *T*5 = no. 72 does not halt for standard starting configuration.

Hold out no. 73

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_2.

S_2

$110_2 \qquad H = 1_0 10$

$010_2 \qquad H = 0_0 10$

$\left.\begin{matrix}110\\010\end{matrix}\right\}\overline{1}111_2 \qquad H = \left.\begin{matrix}1_0 10\\0_0 10\end{matrix}\right\}\overline{1}111$

$0\overline{1}111_2 \qquad H = 0_3\overline{1}111$

Apply the method of exits to $T5$.

exit at X_0

1_0

$1_0 10\overline{11} \qquad 0_0 10\overline{11}$

$0:L \qquad 0:L$

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 73 does not halt for standard starting configuration.

Hold out no. 75

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particular interested in the criteria for S_0 and S_3.

S_0		S_3	
$0_0 0$	$H = 10_1$	$0_3 0$	$H = 10_1$
$0_0 \bar{1}$	$H = 1\bar{1}0_1$	$0_3 \bar{1}0$	$H = 1\bar{1}0_1$
$1_0 0$	$H = 10_4$		

Apply the method of exits to $T5$.

exit at X_4
0_4
10_4 required
$0_2 10$
$01_3 0$ $0 : L$
END
no path

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$, which is not among the halting configurations. Hence, $T5$ = no. 75 does not halt for standard starting configuration.

Hold out no. 76

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_0.

S_0

$0_0\bar{1}0$	$H = 1\bar{1}0_1$
0_00	$H = 10_1$
10_0	$H = 1_30$
001_0	$H = 0_401$
101_0	$H = 1_001$

Apply the method of exits to $T5$.

exit at X_4

1_4

1_401 required (RQ)

101_0

1011_2

10110_4	10111_3	10110_1
10110_401 RQ	10111_30 RQ	1010_00
1011001_0	101110_0	$1:L$
10110011_2	101101_2	
LOOP	LOOP	

The halting configurations are

$$101\begin{cases} \overline{1001}\begin{cases} 1_2 \\ 10_401 \\ 1001_0 \end{cases} \\ \overline{10}\begin{cases} 11_3 \\ 0_2 \\ 1_2 \end{cases} \end{cases}$$

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, $T5$ = no. 76 does not halt for standard starting configuration.

Hold out no. 77

$T4 = T5 - Q2$. Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criterion for S_0.

S_0	
$0_0\bar{1}0$	$H = 1\bar{1}0_1$
$0_0 0$	$H = 10_1$
$011_0\bar{1}0$	$H = 011\bar{1}0_1$
$011_0 0$	$H = 0110_1$
$0\bar{1}11_0\bar{1}0$	$H = 011\bar{1}0_1$
101_0	$H = 1_4 01$
$10\bar{1}1_0$	$H = 10\bar{1}1_0$
001_0	$H = 0_4 01$
$00\bar{1}1_0$	$H = 0_4 01$

Apply the method of exits to *T*5.

exit at X_4

1_4

$1_4 01$ required (RQ)

101_0

1011_2

$0:R$	10110_4	10110_1				
	$10110_4 01$ RQ	$10_0 110$		$011_0 0$		
	$10100\bar{1}1_0$	$101_2 10$		$0:R$	$\boxed{0110_1}$	0110_4
	$101100\bar{1}11_2$	$0:L$	$10_2 110$			$0110_4 01$ RQ
	LOOP		$0:L$			011001_0
						$1:L$

$\boxed{0110_1}$			
$01_2 10$			
$0_2 110$			
$\overline{0_2}\,0110$			
$\bar{0}0_1\bar{0}110$		$\bar{0}0_4\bar{0}110$	
$110_1\bar{0}110$ RQ		$\bar{0}0_4 0110$ RQ	
$0_0 10\bar{0}110$	$011_0 0\bar{0}110$	$\bar{0}001_0 10$	
$01_2 0\bar{0}110$			$1:L$
LOOP			

Loop configurations

$$10\overline{1100\bar{1}}\begin{cases}1_0\\ 11_2\end{cases}$$

$$\left.\begin{matrix}0_0 1\\ 01_2\\ 0_2 1\\ 0_1\bar{0}1\\ \overline{0_2}\,01\end{matrix}\right\}\overline{0\bar{0}110}$$

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, *T*5 = no. 77 does not halt for standard starting configuration.

Hold out no. 78

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_0.

S_0
$00_0\bar{1}1 \quad H = 11\bar{1}1_1$
$0_00 \quad H = 10_1$
$101_0 \quad H = 1_401$
$10_0 \quad H = 1_30$

Apply the method of exits to $T5$.

exit at X_4
1_4
1_411 required
101_0
1011_2

$0:R$	10111_3	10110_1
	10111_30	1010_00
	101110_0	$1:L$
	1011101_2	
	LOOP	

The loop configurations are

$$10\overline{111}01_2 \qquad 10\overline{111}0\begin{cases}110_1\\10_00\end{cases}$$

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 78 does not halt for standard starting configuration.

Hold out no. 79

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_2.

S_2

$$\left.\begin{matrix}0\\1\end{matrix}\right\}011_2 \qquad H = \left.\begin{matrix}0_4\\1_4\end{matrix}\right\}011$$

$$\left.\begin{matrix}0\\1\end{matrix}\right\}0\overline{11}11_2 \qquad H = \left.\begin{matrix}0_4\\1_4\end{matrix}\right\}0\overline{11}11$$

$$011_0 \qquad H = 0_0 11$$

$$0\overline{11}11_0 \qquad H = 0_0\overline{11}11$$

Apply the method of exits to $T5$.

exit at X_0

1_4

$1_4 0\overline{11}11$ required

$10\overline{11}11_2$

$\color{red}{0:1}$ $\quad 10_1\overline{11}11$ $\quad 10\overline{11}1100_4$

$\quad\quad \color{red}{0:1}$ $\quad 10\overline{11}1100_4 0\overline{11}11$ required

$10\overline{11}11000\overline{11}11_2$

$10\overline{11}11000\overline{11}110_1$

LOOP

The loop configuration is

$$1\left\{\overline{10\overline{11}11000\overline{11}11}\right\}\begin{cases}10\overline{11}1100_4 0\overline{11}11\\10\overline{11}11000\overline{11}11_2\\0_1\\10_1\overline{11}110\end{cases}$$

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 79 does not halt for standard starting configuration.

Hold out no. 80

$T4 = T5 - Q2$. Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criteria for S_0 and S_3.

S_0		S_3	
$0_0\bar{1}0$	$H = 1\bar{1}0_1$	$0_30\bar{1}0$	$H = 01\bar{1}0_1$
1_0	$H = 1_0$	$1_310\bar{1}0$	$H = 101\bar{1}0_1$
		1_311	$H = 101_0$
		1_30	$H = 10_4$

Apply the method of exits to *T*5.

exit at X_4

1_4

10_4 required

1_30

0_210

$0:L$	01_00		
	01_00	101_00	
	$1:L$	1_3110	
		0_21110	
		$0:L$	01_0110
			LOOP

The halting loop configuration is

$$\left.\begin{matrix} 0_21 \\ 01_0 \\ 1_3 \end{matrix}\right\}\overline{11}0$$

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, *T*5 = no. 80 does not halt for standard starting configuration.

Hold out no. 81

$T4 = T5 - Q3$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_2 and S_4.

S_2		S_4	
0_2	$H = 0_2$	0_0	$H = 0_0$

Apply the method of exits to $T5$.

exit at X_4		
X_4	0_4	
S_4	0_4	$S_4 = X_4$
	$1_3 0$	
	$0_2 10$	$X_2 = S_2$
	$0:1$	

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$, which is not among the halting configurations. Hence, $T5$ = no. 81 does not halt for standard starting configuration.

Hold out no. 82

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_2.

$$S_2$$

$$\left.\begin{array}{l} 01_2 1\overline{11}0 \\ 0\overline{11}1_2 1\overline{11}0 \\ 0\overline{11}1_2 10 \end{array}\right\} \qquad H = 01\overline{0}10_4$$

$$01_2 \qquad H = 0_0 1$$

$$0\overline{11}1_2 \qquad H = 0_0 \overline{11}1$$

Apply the method of exits to $T5$.

exit at X_4

$X_4 = 0_4$

$X_4 = 01\overline{0}10_4$ required

$01_2 1\overline{11}0$	$0\overline{11}1_2 1\overline{11}0$	$0\overline{11}1_2 10$
$0 : L$	$0 : L$	$0 : L$

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$, which is not among the halting configurations. Hence, $T5$ = no. 82 does not halt for standard starting configuration.

Hold out no. 83

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_3.

S_0		S_3	
$0_0\bar{1}0$	$H = 1\bar{1}0_1$	$10_3\bar{1}0$	$H = 11\bar{1}0_1$
0_00	$H = 10_1$	$00_3\bar{0}10\bar{1}0$	$H = 00\bar{0}11\bar{1}0_1$
1_0	$H = 1_0$	$00_3\bar{0}100$	$H = 0\bar{0}110_1$
		10_30	$H = 110_1$
		$00_3\bar{0}11$	$H = 00\bar{0}11_0$
		00_311	$H = 0011_0$

Apply the method of exits to $T5$.

exit at X_3

1_3

10_2

100_1	101_0
END	END
10_1 required	11_0 required

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, $T5$ = no. 83 does not halt for standard starting configuration.

Loop configurations

$$10\overline{1100\bar{1}}\begin{cases}1_0\\11_2\end{cases}$$

$$\left.\begin{array}{l}0_01\\01_2\\0_21\\0_1\bar{0}1\\\overline{0_2}\,01\end{array}\right\}\overline{0\bar{0}1}10$$

Hold out no. 84

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_3.

S_0		S_3	
$0_0\bar{1}0$	$H = 1\bar{1}0_1$	1_3	$H = 1_3$
0_00	$H = 10_1$	110_3	$H = 1_010$
1_0	$H = 1_0$	$00_3\bar{0}1\bar{1}0$	$H = 00\bar{0}1\bar{1}0_1$

Apply the method of exits to $T5$.

exit at X_3

X_3	1_3		
S_3	1_3	$S_3 = X_3$	
	11_2		
	110_1	111_0	
	10_00	1110_2	
	$1:L$	11101_0	11100_1
		111011_2	
		LOOP	

The loop configuration is

$$1_3 \qquad 11_2 \qquad \overline{11}\begin{cases} 10_2 \\ 0_00 \\ 1_0 \\ 1_2 \\ 0_1 \end{cases}$$

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, $T5$ = no. 84 does not halt for standard starting configuration.

Hold out no. 85

$T4 = T5 - Q4$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_2.

S_0		S_2	
1_0	$H = 1_0$	0_2	$H = 0_2$
$0_0 00$	$H = 1_0 11$		
$0_0 \bar{1}00$	$H = 1_0 \bar{1}11$		
$0_0 \bar{1}01$	$H = 1_0 \bar{1}11$		
$0_0 01$	$H = 1_0 11$		

Apply the method of exits to $T5$.

exit at X_2

X_2	0_2	
S_2	0_2	$S_2 = X_2$
	$0_4 0$	
	$1_3 0$	$1 : L$
	END	
	$1_3 \bar{1}11$ or $1_3 11$ required	

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$, which is not among the halting configurations. Hence, $T5$ = no. 85 does not halt for standard starting configuration.

Hold out no. 86

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_3 and S_4.

S_3		S_4	
0_3	$H = 0_3$	$0_4 0$	$H = 00_1$
11_3	$H = 1_0 1$		
$01_3 \bar{1}0$	$H = \bar{1}10_1$		

Apply the method of exits to $T5$.

exit at X_3

X_3	0_3	
S_3	0_3	$S_3 = X_3$
	01_2	
	00_1	$1:0$
	END	
	no path	

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$, which is not among the halting configurations. Hence, $T5$ = no. 86 does not halt for standard starting configuration.

Hold out no. 88

$T4 = T5 - Q3$. Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criterion for S_4. Apply the method of exits to *T*5. The tree is finite (closes) and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$. Hence, *T*5 = no. 88 does not halt for standard starting configuration.

<u>Heuristic</u>

When selecting a state to remove to obtain a smaller machine, the solution is easier if one removes a state that (a) preserves a connected graph, but (b) disrupts the "flow" of the program significantly.

Hold out no. 89

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_0.

S_0
0_0 $H = 0_0$
011_0 $H = 0_0 11$
$0\overline{11_0}$ $H = 0_0\overline{11}$
$01_0 10$ $H = 0100_3$
$01_0 0$ $H = 010_4$
$0\overline{1_0 1}10$ $H = 0\overline{10}10_4$

Apply the method of exits to $T5$.

exit at X_3
X_3 0_3
0100_3 required
$01_0 10$
$0 : L$

The tree is finite and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$, which is not among the halting configurations. Hence, $T5$ = no. 89 does not halt for standard starting configuration.

Hold out no. 90

$T4 = T5 - Q1$. Note, no. 90 is similar to no. 89. Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criterion for S_0.

S_0
$0_0 \quad H = 0_0$
$011_0 \quad H = 0_0 11$
$0\overline{11_0} \quad H = 0_0\overline{11}$
$01_0 10 \quad H = 0100_3$
$01_0 0 \quad H = 010_4$
$0\overline{1_0}110 \quad H = 0\overline{10}10_4$

Apply the method of exits to *T*5. This generates a tree for the exit at X_3. The tree is finite, but larger than the norm for five-state machines, and we do not give it here. It has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0_0}}$, which is not among the halting configurations. Hence, *T*5 = no. 90 does not halt for standard starting configuration. (To halt, the tape must have a pre-existing 10 to the right of the scanned position.)

To confirm this, we may examine also the effect of the input of the standard starting configuration.

$S_0 = \overline{\overline{0_0}}$
1_1
10_1
$11_0 = 011_0$
$0_0 11$
$1_1 11$
1110_1
$1111_0 = \overline{11_0}$
$0_0\overline{11}$
$1\overline{11}$
$1\overline{11}0_1$
$\overline{11}0_0$
LOOP

The program has entered an infinitely repeating loop.

Hold out no. 91

$T4 = T5 - Q2$. Apply the method of exits to *T*4 and write its complete criterion. We are particularly interested in the criteria for S_0 and S_3.

S_0		S_3	
1_0	$H = 1_0$	$1_3 1$	$H = 11_4$
		0_2	$H = 0_3$

Apply the method of exits to *T*5.

exit at X_3			
0_3			
$0_2 0$			
$1_4 0$	$1:L$		
$11_4 0$ required			
$1_3 10$			
$0_2 110$			
$1:0$	$1_4 110$	$01_0 10$	
	$11_4 110$	$011_2 0$	
	$1_3 1110$	$1:0$	$1:L$
	$0_2 11110$		
	LOOP		

The halting configurations are

$$\left.\begin{matrix} 11_4 \\ 1_3 1 \\ 0_2 \end{matrix}\right\} \overline{11}0 \qquad 0\overline{11_2}\,0 \quad 0\overline{1_0 1}0$$

The tree is finite, but larger than the norm for five-state machines, and we do not give it here. It has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$, which is not among the halting configurations. Hence, *T*5 = no. 91 does not halt for standard starting configuration.

Hold out no. 92

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_3.

S_0		S_3	
1_0	$H = 1_0$	1_3	$H = 1_3$
10_1	$H = 1_0 0$		
$0_0\bar{1}0$	$H = 1\bar{1}0_0$		
$0_0 0$	$H = 10_0$		

Apply the method of exits to $T5$.

exit at X_3

X_3	1_3	
S_3	1_3	$S_3 = X_3$
	10_2	
	101_0	
	1011_2	
	10111_0	
	LOOP	
	$10\overline{11}1_0$	

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, $T5$ = no. 92 does not halt for standard starting configuration.

Hold out no. 93

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_3 and S_4.

S_3		S_4	
1_3	$H = 1_3$	11_4	$H = 1_01$
110_3	$H = 1_010$	$01_4\bar{1}10$	$H = 0\bar{1}1_010$
$00_3\bar{0}01\bar{1}10$	$H = \bar{0}\bar{1}1_010$	$0_41\bar{1}10$	$H = 0\bar{1}1_010$
		$0_4\bar{0}01\bar{1}10$	$H = \bar{0}\bar{1}1_010$

Apply the method of exits to $T5$.

exit at X_3

X_3	1_3		
S_3	1_3	$S_3 = X_3$	
	10_2		
	101_0		
	101_010		101_01
	10110_3		1011_4
	101100_2		10111_2
	1011001_0		101111_0
	LOOP		LOOP

The halting configurations are

$$10\begin{Bmatrix}\overline{1100}\\ \overline{111}\end{Bmatrix}\begin{cases}1_010\\ 1_01\\ 110_3\\ 100_2\\ 11_4\\ 111_2\end{cases}$$

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, $T5$ = no. 93 does not halt for standard starting configuration.

Hold out no. 94

$T4 = T5 - Q2$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criteria for S_0 and S_3.

S_0		S_3	
1_0	$H = 1_0$	1_3	$H = 1_3$
$0_0 0$	$H = 1_0 0$		
$0_0 \bar{1} 0$	$H = \bar{1} 1_0 0$		

Apply the method of exits to $T5$.

exit at X_3

X_3	1_3		
S_3	1_3	$S_3 = X_3$	
	10_2		
	101_0		
	101_0		$101_0 0$
	1011_2		$100_0 0$
	10111_0		1000_1
	LOOP		LOOP

The halting configurations are

$$10\left\{\overline{11}\right\}\begin{cases} 1_0 \\ 11_2 \\ 1_0 0 \\ 0_0 0 \\ 00_1 \end{cases}$$

The standard starting configuration $S_0 = \overline{\overline{0}}_0$ is not a halting configuration. Hence, $T5$ = no. 94 does not halt for standard starting configuration.

Hold out no. 95

$T4 = T5 - Q3$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_4. Apply the method of exits to $T5$. $T5$ closes almost immediately.

exit at X_3
1_3
$0_2 1$ $1:0$
$0_4 1$
$0:L$

The tree is finite (closes) and has no backward trace to the standard starting configuration, $S_0 = \overline{\overline{0}}_0$. Hence, $T5$ = no. 95 does not halt for standard starting configuration.

Hold out no. 96

$T4 = T5 - Q1$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_2.

S_2
$0_2 1 \quad H = 01_0$
$0_2 0 \quad H = 01_0$
$111_2 \quad H = 1_4 11$
$1\overline{10}11_2 \quad H = 1_4 \overline{10}11$

Apply the method of exits to $T5$.

exit at X_4
1_4
$1_4 11$ required
111_2
110_1
$11_0 0$
END
01_0 required

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 96 does not halt for standard starting configuration.

Hold out no. 97

$T4 = T5 - Q4$. Apply the method of exits to $T4$ and write its complete criterion. We are particularly interested in the criterion for S_0.

S_0

11_0	$H = 1_1 1$
10_0	$H = 1_1 10$
$0\overline{01}01_0 0$	$H = 0_3 \overline{01}011$
$001_0 0$	$H = 0_3 011$
$000_0 0$	$H = 0_3 011$

Apply the method of exits to $T5$.

exit at X_4

1_4

$1:0$ $0_3 1$

END

$0_3 011$ required

The standard starting configuration $S_0 = \overline{\overline{0_0}}$ is not a halting configuration. Hence, $T5$ = no. 97 does not halt for standard starting configuration.

Index of hold outs

The numbering of the hold outs is defined in Kellett [2005]. Here the term "text" refers to the worked examples in the paper, *Solution to the Halting Problem*, Melampus 2019.