Absolute and relative errors

Absolute error

All physical measurements are not exact. They are measured to a degree of accuracy. For example, 3.14 is a measurement that could correspond to any real value between 3.135 and 3.145. In this instance the absolute error is ± 0.5.

The absolute error is the magnitude of the real value – the approximate value

$$\text{Absolute error} = \left|\text{Real value} - \text{Approximation}\right|$$

For example, if the real value is 3.137 and the approximation is 3.14 then

$$\begin{aligned} \text{Absolute error} &= \left|3.137 - 3.14\right| \\ &= \left|-0.003\right| \\ &= 0.003 \end{aligned}$$

When physical quantities are combined through multiplication and division, the error can be magnified. For example, the largest value when 3.14 is divided by 2.67 is the greatest value of 3.14 divided by the least value of 2.67. That is,

$$\frac{3.145}{2.665}$$

We can be asked to find the size of the error given the real value, or the range of possible values.

Relative error

The relative error is the error divided by the real value and expressed as a percentage.

$$\text{Relative error} = \frac{\text{Absolute error}}{\text{Real value}} \times 100\%$$

<u>Example</u>

One of the exact roots of a certain quadratic equation is $\dfrac{2+\sqrt{3}}{2}$.

For this root, find, correct to 1 significant figure, the magnitude of the relative error that arises if the value used for $\sqrt{3}$ is rounded 1 decimal palace.

Solution

$\sqrt{3} = 1.7 \quad$ (1.D.P)

$\frac{2+1.7}{2} = 1.85$ whereas $\frac{2+\sqrt{3}}{2} = 1.866...$

Absolute error: $1.866 - 1.85 = 0.0160...$

$$\text{Relative error} = \frac{\text{absolute error}}{\text{real value}} = \frac{0.0160...}{1.866...} = 0.00858... \approx 0.9\% \text{ (1.S.F)}$$