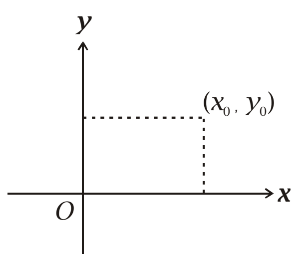
Cartesian Coordinates

Lengths

A *point* in two dimensions is specified by an *ordered pair* of numbers, (x_0, y_0),where x_0 stands for the *x*-coordinate of the point, and y_0 for the corresponding *y*-coordinate. The expression x_0 may be your first introduction to the use of *subscripts* to represent a value, and is read "*x* sub 0".

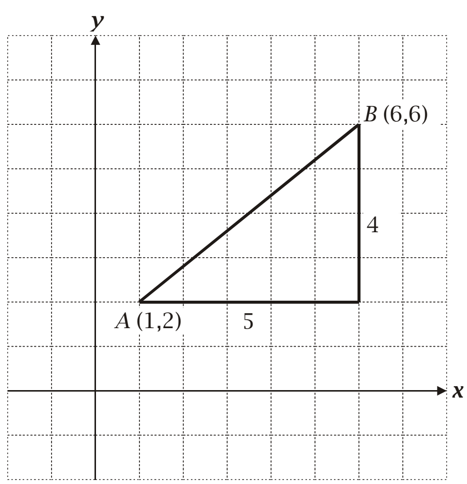


To find the distance between two points, $A = (x_0, y_0)$ and $B = (x_1, y_1)$, begin by sketching the positions of the two points. Join up the two points, and form a right-angled triangle with the line as the hypotenuse. Then use Pythagoras's Theorem to find the length of the hypotenuse.

Example (1)

Find the length of the line joining $A(1,2)$ to $B(6,6)$.

Solution



The length of *AB* is given by $|AB| = \sqrt{5^2 + 4^2} = \sqrt{25+16} = \sqrt{41}$

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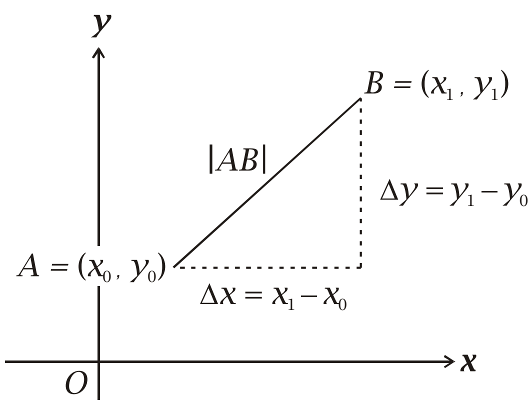
This answer uses the symbol $|AB|$ to represent a length. This symbol (or similar) is used throughout higher level mathematics, but it is useful rather than essential at this level. You may also use the expression *AB* or even just "length" or "distance". If we wish to express the distance between two points in algebraic symbols, this would be

$|AB| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$.

This follows from Pythagoras's theorem, and is illustrated by the following diagram.



The above diagram also introduces the symbols Δx and Δy in

$\Delta x = x_1 - x_0 \qquad \Delta y = y_1 - y_0$.

Here Δx and Δy can be read "change in *x*" and "change in *y*" respectively. These symbols $(\Delta x, \Delta y)$ are not essential but it is very useful to recognise them from an early stage and even say in your head "change in *x*" when you see Δx. This will facilitate problem solving immediately and understanding of subsequent concepts. Such practice will also prevent certain errors from creeping in at a later stage. Using these symbols the distance formula becomes $|AB| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$.

Mid-points

The *mid-point* of the line joining *A* to *B* is

$$M = (\bar{x}, \bar{y}) = \left(\frac{x_1 + x_0}{2}, \frac{y_1 + y_0}{2} \right)$$

Add the two coordinates separately, and divide each sum by 2. In other words, the mid-point is the *average* of the coordinates of the separate points. The symbol $\bar{x}$ is used throughout mathematics to denote an average, so it can be used here as well.

Example (2)

Find the mid-point of $A(-1,4)$ and $B(3,7)$

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Solution

The average of the x-coordinates is $\frac{-1+3}{2}=\frac{2}{2}=1$

The average of the y-coordinates is $\frac{4+7}{2}=\frac{11}{2}$

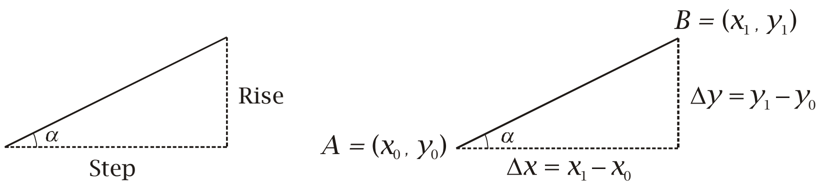
Hence the mid-point is $M=(\bar{x},\bar{y})=\left(1,\frac{11}{2}\right)$

Gradients

The gradient of a line joining the points

$A=(x_0,y_0)$ and $B=(x_1,y_1)$

is its slope, which is given by the "rise" over the "step".



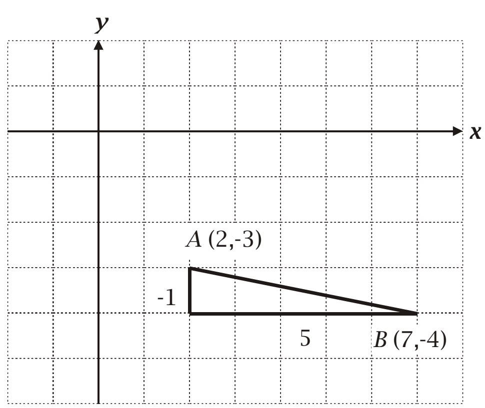
The gradient is often denoted by m and it is the same as the tangent of the angle made by the line with the x-axis. We again introduce the symbol Δ to denote "change in" or "increase in".

$$m=\frac{\Delta y}{\Delta x}=\frac{\text{change in } y}{\text{change in } x}=\frac{y_1-y_0}{x_1-x_0}$$

Example (3)

Find the gradient of the line joining the points $(2,-3)$ and $(7,-4)$

Solution



$$m=\frac{\Delta y}{\Delta x}=\frac{y_1-y_0}{x_1-x_0}=\frac{-4-(-3)}{7-2}=-\frac{1}{5}$$

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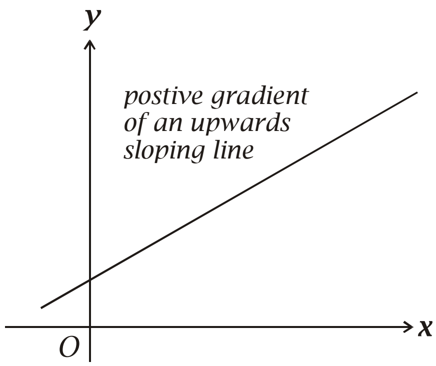
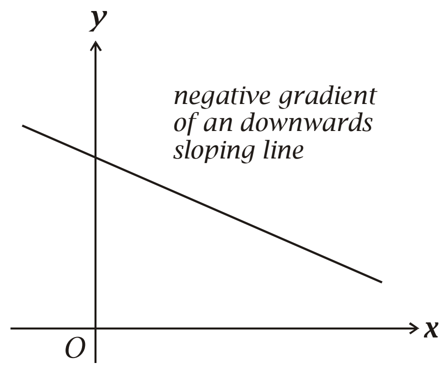
In this last example the line in question was sloping downwards and its gradient was negative. The meaning of positive and negative gradients is shown by the following diagrams.





Therefore it is important that when substituting into the formula

$$m = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_1 - y_0}{x_1 - x_0}$$

that the points should be substituted consistently.