Centre of Mass of a Composite Body

A composite body is one made up of separate parts each of identifiable shape and mass and each having its own centre of mass. The aim is to find the centre of mass of the composite body. This is done by means of taking moments according to the principle that the moment of the whole body about its centre of mass is equal to the sum of the moments of each parts.

Let *K* be a composite body comprising *n* parts of mass M_i, $0 \le i \le n$. Let

$$M = \sum m_i$$

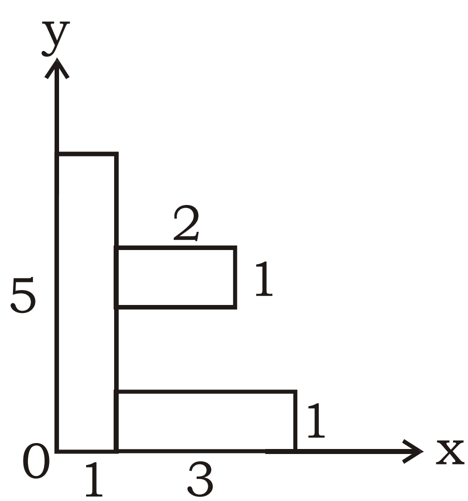
be the total mass of *K*. Let *P* be any turning point or *L* be any axis. Let x_i be the distance of the centre of mass m_i from *K* or the perpendicular distance of the centre of mass m_i from *L*. Let $\overline{x}$ be the perpendicular distance of *M* from *P* or *L* of the centre of mass of *K*. Then

$$M\overline{x} = \sum m_i x_i$$

We proceed to apply this result to a number of examples.

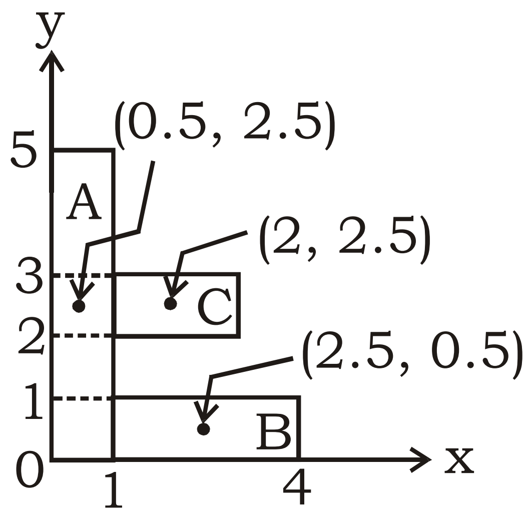
<u>Example (1)</u>

A lamina is made up in the shape of the symbol as shown:



Find its centre of mass.

Let $(\overline{x}, \overline{y})$ be the centre of mass. The coordinates of the centre of mass of each piece are as indicated.



Labelling the segments as shown, their masses are $m_A = 5$ $m_B = 3$ $m_C = 2$ units of mass. Total mass is $M = m_A + m_B + m_C = 5 + 3 + 2 = 10$.

Then, taking moments about *Oy*

$$\begin{aligned} 10\overline{x} &= m_A x_A + m_B x_B + m_C x_C \\ &= (5 \times 0.5) + (3 \times 2.5) + (2 \times 2) \\ &= 14 \end{aligned}$$

$$\overline{x} = 1.4$$

Taking moments about *Ox*

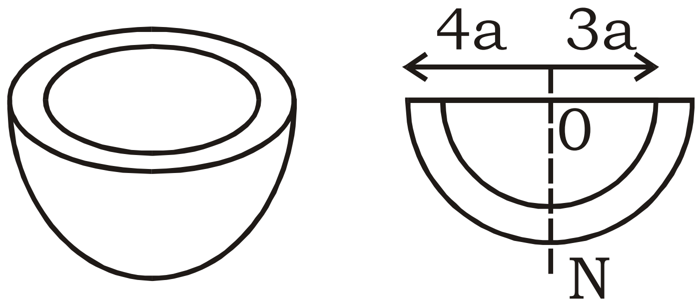
$$\begin{aligned} 10\overline{y} &= m_A y_A + m_B y_B + m_C y_C \\ &= (5 \times 2.5) + (3 \times 0.5) + (2 \times 2.5) \\ &= 19 \end{aligned}$$

$$\overline{y} = 1.9$$

You can be asked to find the centre of mass created by the removal of one solid shape from another. The following example illustrates this type of problem and how it is solved.

Example (2)

A wooden bowl is made by removing a hemispherical portion of radius $3a$ from solid hemisphere of wood of radius $4a$.



Given that the centre of mass of a uniform solid hemisphere of radius *r* is 3*r*/8 from its plane face, find the centre of mass of the bowl from the origin, *O*, along the axis *ON*. When the bowl is suspended from its rim its diameter makes an angle α with *ON*. Find α. Let *x*-bar be the position of the centre of mass of the bowl from *O* along the axis *ON*.

The volume of a hemisphere of radius *r* is

$$\frac{2}{3}\pi r^3$$

Hence the volume of solid hemisphere of radius 4*a* is

$$\frac{2}{3}\pi(4a)^3 = \frac{128\pi a^3}{3}$$

And the volume of the cavity is

$$\frac{2}{3}\pi(3a)^3 = 18\pi a^3$$

Let ρ be the density of the wood per unit volume. Then the mass of the solid hemisphere is

$$\frac{128\pi a^3\rho}{3}$$

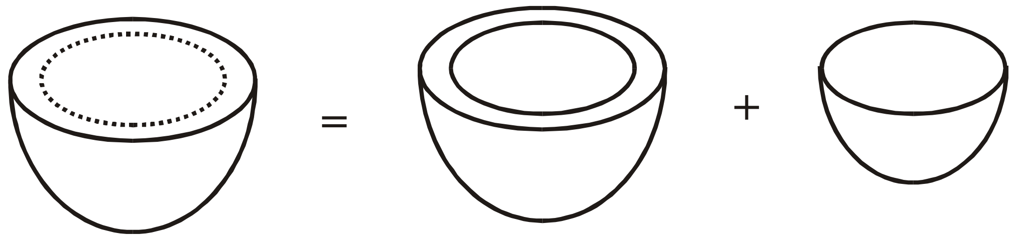
The missing mass of the cavity is

$$18\pi a^3\rho$$

The mass of the bowl is

$$\pi\rho a^3\left(\frac{128}{3} - 18\right) = \frac{74}{3}\pi\rho a^3$$

The solid hemisphere can be viewed as made of two parts: the bowl and the smaller solid hemisphere removed from it.



Large solid hemisphere = Bowl + small solid hemisphere

A = B + C

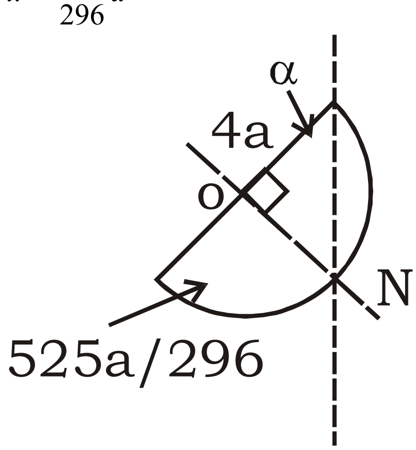
$$\therefore M_A X_A = M_B X_B + M_C X_C$$

$$\therefore \left(\frac{128\pi a^3 \rho}{3}\right)\times\left(\frac{3(4a)}{8}\right) = \frac{74\pi\rho a^3}{3}\bar{x} + 18\pi\rho a^3 \times\left(\frac{3(3a)}{8}\right)$$

$$\frac{74\bar{x}}{3} = 64a - \frac{81}{4}a$$

$$\frac{74\bar{x}}{3} = \frac{175}{4}a$$

$$\bar{x} = \frac{525}{296}a$$



$$\tan\alpha = \frac{\frac{525}{296}a}{4a} = \frac{525}{1184}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{525}{1184}\right) = 23.9° \quad (0.1°)$$