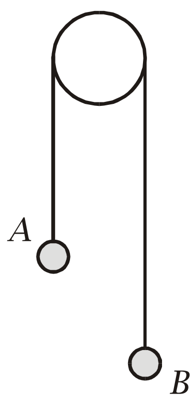
Connected Particles

Two bodies joined together by a cord or similar means are called *connected particles*. The tension in the cord is constant. When free to move, they move together. The size of the acceleration of the two particles is also the same for both particles. Because the two particles are connected together they are called a *system*. Typically in questions, you are given the masses of both particles and you are asked to find their common acceleration and the tension in the cord. By this stage you should be already familiar with all the theory required in order to solve problems involving connected particles. The essential prerequisite knowledge required is understanding of Newton's second law and of resultant forces.

Example (1)

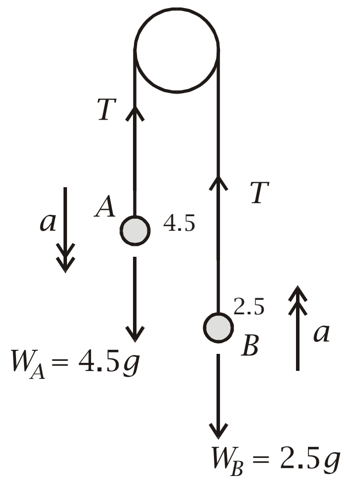
The diagram shows two particles A and B, of mass 4.5 kg and 2.5 kg respectively, connected by a light inextensible string passing over a smooth pulley. Initially, B is held at rest with the string taut. It is then released.



Calculate the magnitude of the acceleration of A and the tension in the string.

Solution

We should begin by redrawing the diagram marking the forces onto it. Since A has larger mass than B once the system is released from rest A falls and B rises. The tension T in the cable is the same for both particles, and the acceleration a of both particles is also the same. The acceleration is produced by the resultant forces acting at A and B individually.



In the diagram we have used a double-headed arrow for the acceleration because it is not a force and is produced by the resultant forces acting at *A* and *B* individually. The problem asks us to find two unknown quantities, the tension (*T*) and the acceleration (*a*). This implies that there should be *two* equations involving the two unknown quantities and that we should solve them simultaneously. To find the two equations we resolve forces at *A* and *B* separately. At *A* the resultant force is

$$R = W_A - T$$

where W_A is the weight of *A*. The weight of *A* is $W_A = m_A g$ where $m_A = 4.5\,\text{kg}$ is the mass of *A*. Applying Newton's second law to the resultant and substituting for W_A we get

$$m_A a = m_A g - T\,.$$

Substituting $m_A = 4.5$ and $g = 9.8$

$$4.5a = 4.5 \times 9.8 - T$$
$$4.5a = 44.1 - T \qquad (1)$$

This is the first of the simultaneous equations that we are seeking. To find the second we repeat the analysis for particle *B*. This time the tension *T* is greater than the weight of *B*, W_B, so

$$R = T - W_B$$
$$m_B a = T - m_B g$$

$$2.5a = T - 2.5 \times 9.8$$
$$2.5a = T - 24.5 \qquad (2)$$

We are required to solve simultaneously

$$\begin{cases} 4.5a = 44.1 - T & (1) \\ 2.5a = T - 24.5 & (2) \end{cases}$$

Adding the two equations gives

$$\begin{cases} 4.5a = 44.1 - T & (1) \\ 2.5a = T - 24.5 & (2) \end{cases}$$

$$7a = 19.6$$

$$a = 2.8 \ ms^{-2}$$

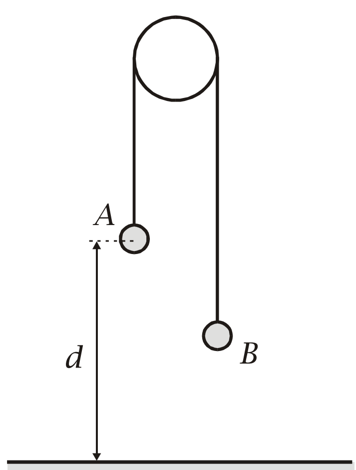
Substituting this value into the first equation gives

$$\begin{aligned} T &= 44.1 - 4.5 \times 2.8 \\ &= 31.5 \text{ N} \end{aligned}$$

The annotation of this solution makes it seem more complicated than it is, so we present a second example with minimal annotation.

Example (2)

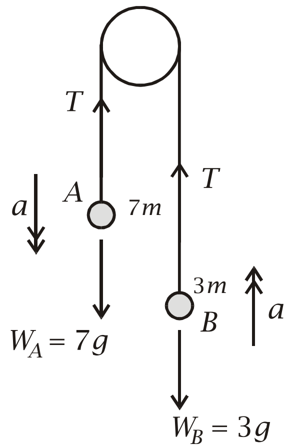
As shown in the diagram, two particles A and B are connected by a light inextensible string which passes over a smooth pulley whose axis is fixed. Particle A is of mass 7 m and particle B is of mass 3 m. Initially B rests on a fixed horizontal plane and A hangs at a height d above the plane. The particles are released from rest in this position with the string taut.



(a) Find the tension in the string and the magnitude of the acceleration.

(b) Given that it takes 1.2 s for particle A to reach the plane, find the height d.

Solution



(*a*) Resolving at *A*

$$m_A a = m_A g - T$$
$$7ma = 7m \times 9.8 - T$$
$$7ma = 68.6m - T \qquad (1)$$

Resolving at *B*

$$m_B a = T - m_B g$$
$$3ma = T - 3m \times 9.8$$
$$3ma = T - 29.4m \qquad (2)$$

Solving simultaneously

$(1)+(2)$ $\quad 10ma = 68.6m - 29.4m$

$$10a = 39.2$$
$$a = 3.92 = 3.9 \ ms^{-2} \ (2.s.f.)$$

Sub in (1) $\quad 7m \times 3.92 = 68.6m - T$

$$T = (68.6 - 27.44)m = 41.16m = 41m \ N \ (2.s.f.)$$

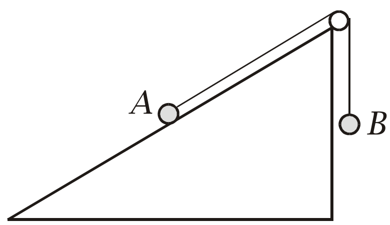
(*b*) This is an application of the equations of uniform acceleration. The acceleration is $a = 3.92$, the initial velocity is $u = 0$, the distance is $s = d$ and the time is $t = 1.2$. The appropriate equation is $s = ut + \frac{1}{2}at^2$ and on substitution we obtain

$$d = \frac{1}{2} \times 3.92 \times (1.2)^2$$
$$= 2.8224$$
$$= 2.8 \text{ m } (2.\text{s.f.})$$

In the following example one particle is at rest on an inclined plane before the system is released. This complicates the algebra because we have to find the component of the weight that acts down the slope and also take friction into consideration. However, there is no new theory involved – it is simply a more involved application of ideas that you should already have met.

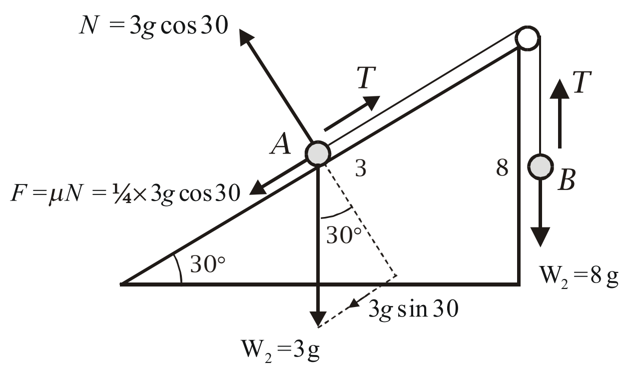
Example (3)

The diagram shows a mass A of $3kg$ lies on a slope of an angle 30° and coefficient of friction $\frac{1}{4}$. It is attached by a light inextensible string over a smooth pulley to a mass B of $8kg$ which hangs freely. Find the acceleration and the tension of the system. of A. The friction is given by the equation $F = \mu N$ where $\mu = \frac{1}{4}$ as given in the question. Take $g = 9.81$.



Solution

The forces acting at A are the weight of A, the tension, the normal reaction and friction. The weight must be resolved into components acting down the slope and perpendicularly to the slope. The normal reaction is equal and opposite to the perpendicular component of the weight



Resolving at A

Resultant force = tension – component of the weight acting down the slope – friction

$m_1 a = T - m_1 g\left(\sin 30° + \mu \cos 30°\right)$

Subsituting

$$m = 3,\ g = 9.81,\ \mu = \frac{1}{4} = 0.25,\ \sin 30° = \frac{1}{2},\ \cos 30° = \frac{\sqrt{3}}{2}$$

Hence

$$a = \frac{T}{3} - 9.81\left(0.5 + 0.25 \times \frac{\sqrt{3}}{2}\right)$$

$$a = \frac{T}{3} - 7.029... \qquad (1)$$

Resolving at B

Resultant force = weight - tension

$$m_2 a = m_2 g - T$$
$$8a = 8g - T$$
$$a = 9.81 - \frac{T}{8} \qquad (2)$$

Solving simultaneously for equations (1) and (2)

$$\frac{T}{3} - 7.029... = a = 9.81 - \frac{T}{8}$$
$$8T - 168.694... = 235.44 - 3T$$
$$11T = 404.134...$$
$$T = 36.739... = 37\,\text{N}\ (2.\text{s.f.})$$

Substituting in (1)

$$a = \frac{T}{3} - 7.029... = 5.2\ \text{ms}^{-2}$$