Decision Trees

Decision making

Decision trees are concerned with a mathematical technique of arriving at a decision. Questions involving decisions often arise in a business context, and involve financial implications.

For example, the director of a company is deciding whether to open a new factory or not. He would like some way of quantifying the decision making process – in other words, he would like some way of translating possible outcomes measured in money terms, and their probabilities into expected outcomes – revenues and costs. Those expected outcomes would then enable him to make the decision.

The future is always uncertain, but a close examination of outcomes and their probabilities, and decisions taken in the light of those, are believed to improve the odds of success. Decision trees are about narrowing the odds, and tipping the balance in our favour.

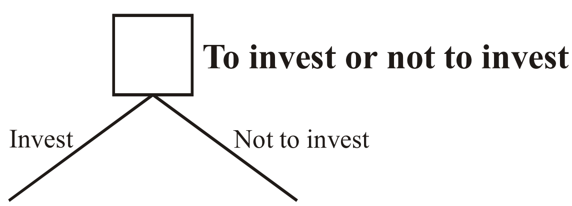
Lines and nodes in a decision tree

Decision trees are tree like structures involving lines and nodes. The nodes are of three types.

<u>Decisions</u>

Decisions are represented by square boxes.

Branches arising from a decision box are labelled with the decision that they represent.

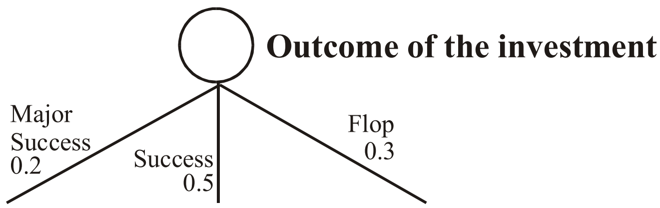


<u>Outcomes</u>

The outcomes of decisions are represented by circles.

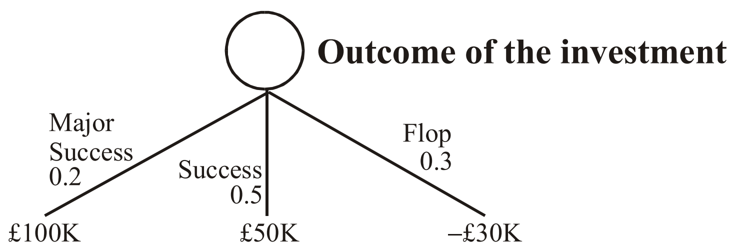
Outcomes can arise from a single possibility, but usually there are several possibilities. Once a decision has been taken, the outcome of that decision usually arises from more than one possible result of the decision. Those possible results are represented as branches running out of the outcome circle. For example, an investment could succeed or fail

The probabilities of those outcomes are also placed against the branches.

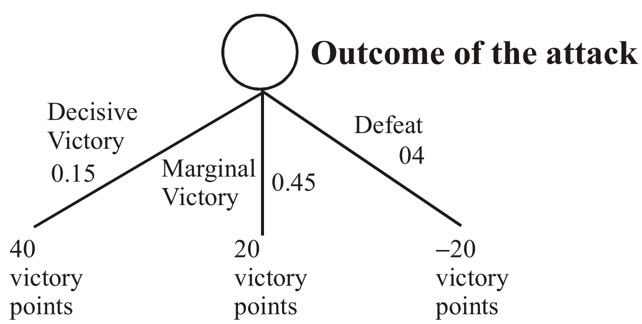


Results

The tree branches end in some kind of result. This is often measured in monetary terms.



Other measures of utility or satisfaction can be used. For example, victory points in a war game.



These are the units by which the outcomes and decisions will be evaluated. They are placed at the final, terminal nodes of the decision tree, and represent the ultimate results in terms of which the decisions taken are evaluated.

The process of evaluating a decision, or series of decisions, is best illustrated through example.

Worked examples

Example (1)

A company is deciding whether to open a new factory or not and hence increase capacity. The increase in capacity will enable the company to sell more of its product, but it cannot be guaranteed that they will be able to sell everything they make. Opening the factory will incur a cost, and the question is whether it will be worth the risk of not selling their product in order to secure the possible additional profits of the increased output. It will cost £1m to open the new factory.

If the factory is opened, then there is a 30% probability of a major marketing success leading to profits of £2m. On the other hand, there is a 60% probability of selling only half the factory's output, leading to profits of £1.2m. Finally, there is a 10% probability that none of the additional output can be sold; hence, profits would be 0.

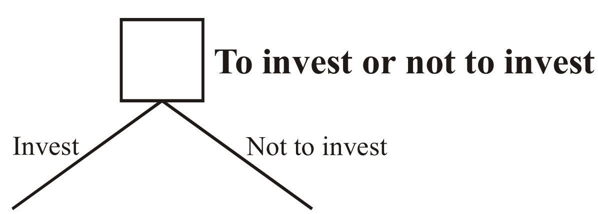
If the factory is not opened, the company will be able to make increased profits of £0.2m from increased output at their existing plants.

Assuming the cost of the factory must be met by these profits, should the company invest or not?

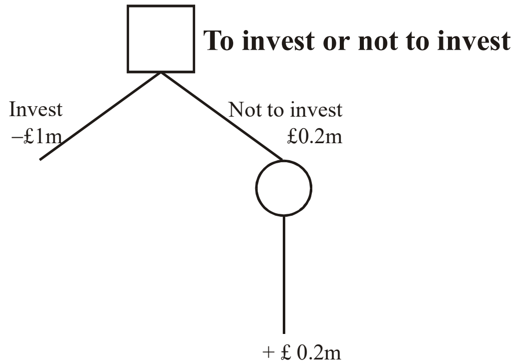
Solution

We begin by drawing a decision tree. We will illustrate this process in stages.

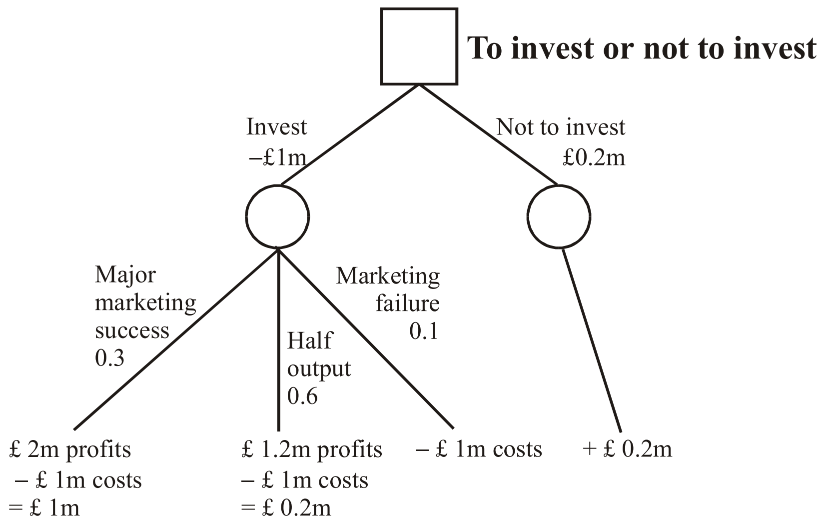
The decision is whether to invest or not.



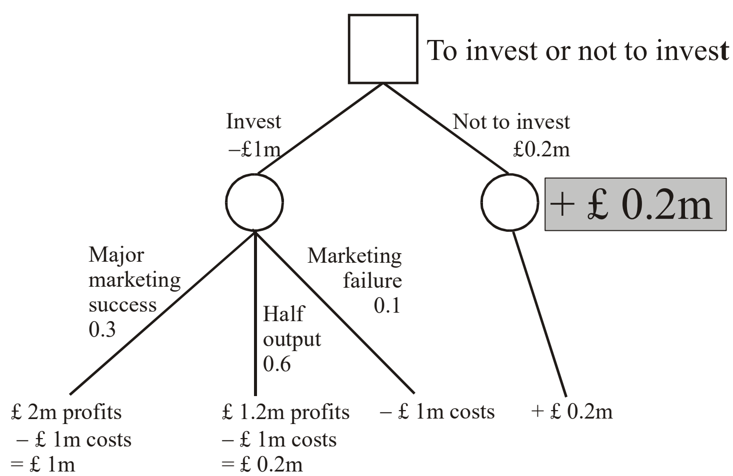
The outcome of not investing is increased profits of £0.2m.



The outcome of the decision to invest depends on three possibilities



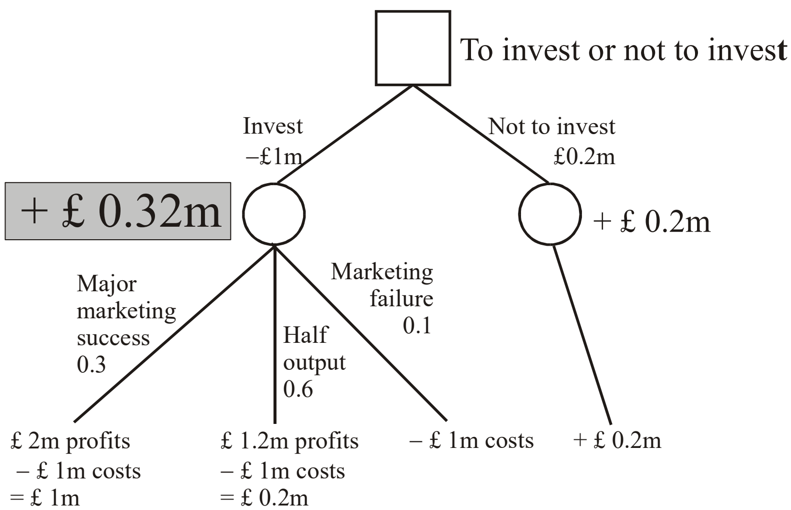
We now evaluate the outcomes. The outcome of the decision not to invest is clearly just + £ 0.2m.



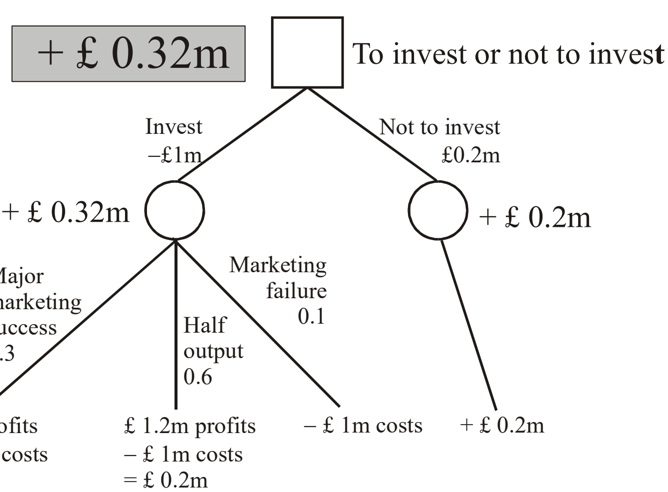
The outcome of the decision to invest depends on three possibilities. The overall outcome is a weighted average of the results of these possibilities calculated on the basis of their probabilities.

$$\text{Outcome} = £\text{m}\{(1\times 0.3)+(0.2\times 0.6)+(-1\times 0.1)\}$$
$$= £0.32\text{m}$$

This value is placed against the circle to which it applies.



The obvious thing to decide is to choose that outcome that has the best possible evaluation. We chose the better of the two values, + £ 0.2m and + £ 0.32m, and place this value next to the appropriate decision node.



So the final solution to this problem is to choose to invest – a decision which is expected to bring in, on average £0.32m as opposed to the decision not to invest, which is expected to bring in £0.2m.

Example (2)

Osmond is a fortune hunter who is pursuing an American heiress, Isabella. If he is successful in marrying her, he will gain her fortune of $750K. Isabella has decided to visit Italy. In order to gain her hand in marriage, and win the fortune, he must propose. However, before he does so, he has two decisions to make. Firstly, he could decide to visit Italy as well, but this will cost him the travelling expenses and the additional expenses in reopening his "chateau" at $100K. If he does visit Italy, he will have to decide whether to invite his daughter, Polly, to the chateau, but this will incur an additional expense of $20K.

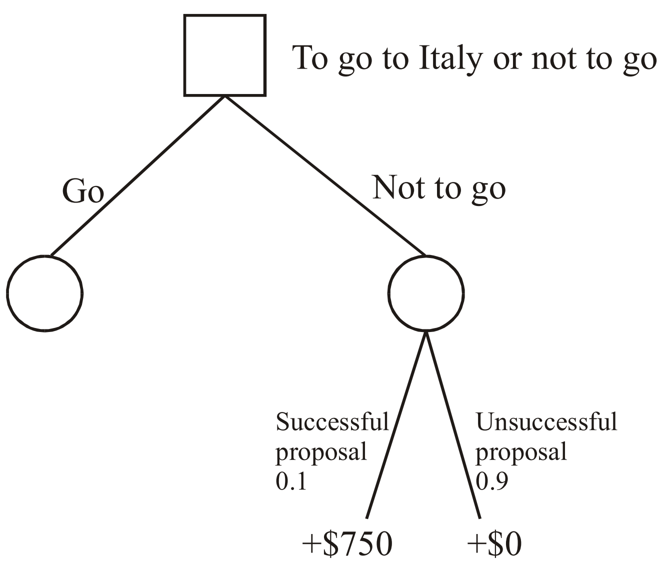
If he decides not to visit Italy then his chances of a successful proposal are 10%. If he decides to visit Italy, then his chances of a successful proposal increase to 50%. Finally, since Isabel will be strongly influenced by a sympathy vote for him if she meets his daughter, his chances of proposing successfully will increase to 60%.

What should he do?

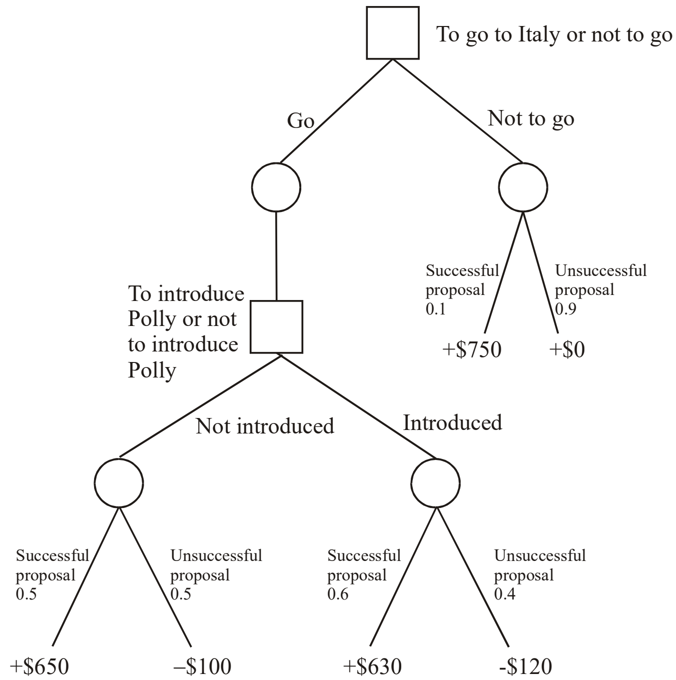
Solution

The first decision he has to take is whether to travel to Italy or not. If he does not go to Italy, then he has no other decision to take.

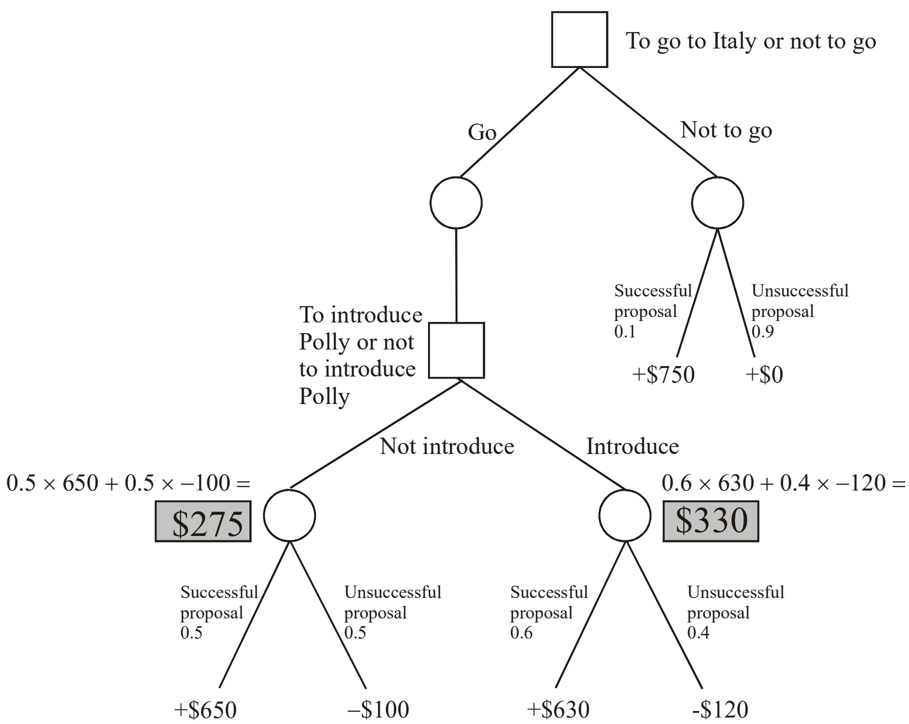
In these diagrams, all monetary values are in $'000s – in thousands of dollars.



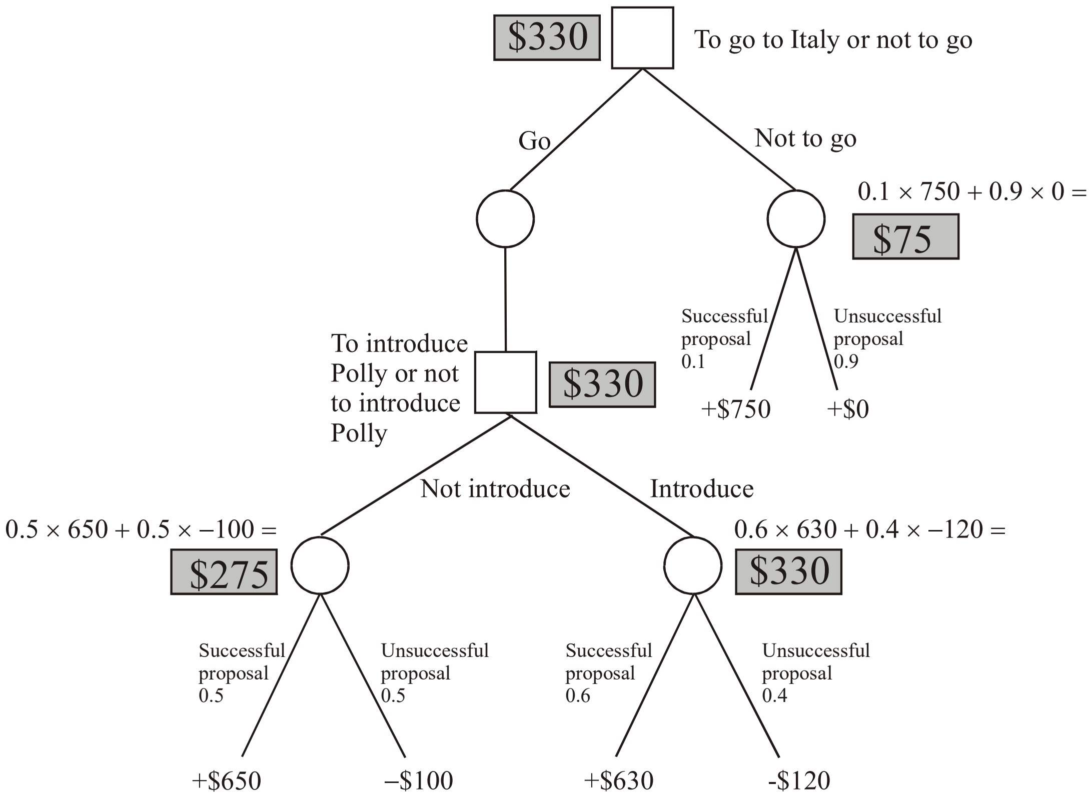
If he does go to Italy, then he must decide whether to introduce his daughter to Isabel.



The outcomes must now be evaluated. Firstly, the outcomes associated with bringing Polly into the scene or not.



On this basis he would decide to introduce Polly to Isabel. The other option of not going to Italy at all must also be evaluated.



So Osmond will decide to go to Italy and introduce his daughter to Isabel. He expects to make $330K out of these decisions.

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