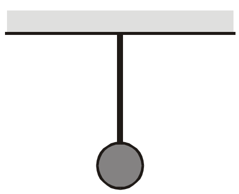
Hooke's Law

Prerequisites

You should be familiar already with static equilibrium.

Example (1)

The diagram shows a sphere of mass 2 kg suspended by means of a light cord to the ceiling running through a smooth hook.

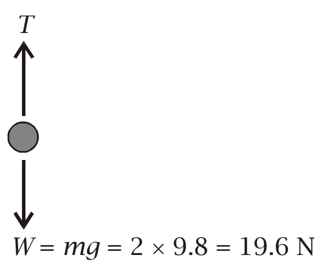


(*a*) Find the tension in the cord.

(*b*) Explain in terms of physical principles how the tension in the cord is produced.

Solution

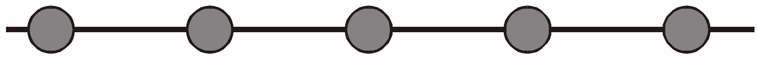
The question uses the term *light*. This is introduced to indicate that we can ignore the weight of the cord. The term *smooth* indicates that the friction at the hook is zero – it too can be ignored.



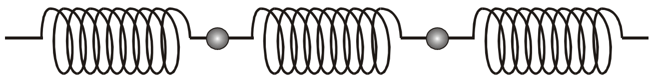
(*a*) Only two forces act on the sphere, its weight and the tension in the cord. Because the sphere is in static equilibrium these two forces are equal in magnitude and opposite in effect. Therefore

$T = W = mg = 2 \times 9.8 = 19.6$ N

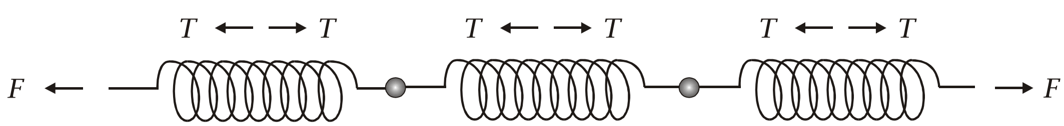
(*b*) It is the stretching of the cord that produces the tension. The cord is made of atoms or molecules. Between these atoms or molecules there are chemical bonds.



Each bond may be thought of as a spring linking one atom (or molecule) to another.



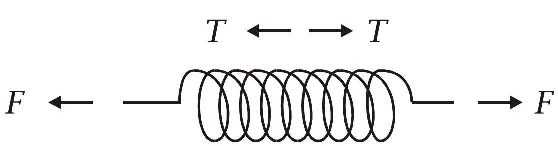
When the sphere is attached to the cord these bonds are stretched. There are forces pulling the cord in both directions. Because the cord is in static equilibrium these forces are equal in magnitude and opposite in direction. These forces cause the bonds to stretch. Then the bonds react *like springs* and as the bonds stretch they start to pull back the atoms (or molecules). Eventually a state of equilibrium is reached when the forces in the spring are equal to the forces applied at both ends.



So the tension in the cord is produced by the stretching of the bonds. The bonds act as springs.

Elasticity

When a spring or piece of *elastic* string is stretched it exerts a force that opposes the stretching.



Imagine pulling the spring from both ends. Experience indicates that a tension pulling the two ends back together is created. We also know from experience that it is only possible to carry on stretching the spring up to a certain limit – at which point the spring resists any further stretching until it deforms and eventually breaks. When the spring is not being stretched it has a *natural* *length* that we denote by l_0. Provided the spring has not been deformed by being stretched too much, when we stop stretching it, it contracts back to its natural length. So the natural length is an intrinsic property of a "healthy" spring. We let l denote the length of the spring when stretched, so the *extension* of the string is $x = l - l_0$. For many springs it is an empirical law of nature that the strength of the force produced by the spring is proportional to the extension. This law is known as *Hooke's law*.

Hooke's law

$F = kx$

where $x = l - l_0$ is the extension of the spring and k is a constant of proportionality known as the *spring constant* or *stiffness*.

Example (2)

A light elastic spring hangs from a fixed point O. When not stretched it has a natural length of 0.5 m. A particle of mass 2 kg is attached to the lower end and it stretches to 0.85 m. Assume that the spring may be stretched up to a length of 1.50 m without deforming.

(*a*) Find the spring constant.

(*b*) Draw a graph to show how the extension of the spring varies with the force applied to it.

(*c*) What will be the extension produced by a mass of 3.5 kg?

(*d*) What can we say, if anything, about the extension produced by a mass of10 kg?

Solution

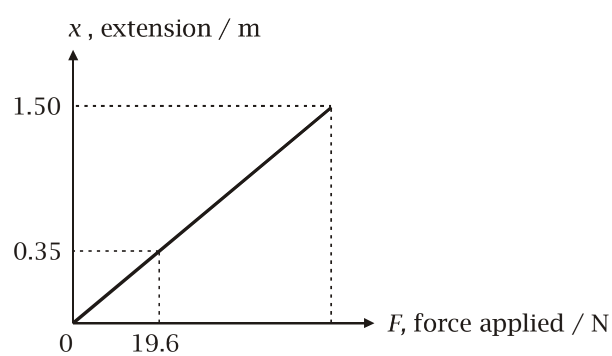
(*a*) Hooke's law is $F = kx$ where $x = l - l_0$ and l_0 is the natural length of the spring.

Then $k = \dfrac{F}{l - l_0}$. Here the force, F, is produced by the weight of the 2 kg mass.

$F = W = mg = 2 \times 9.8 = 19.6$ N and the extension is $x = l - l_0 = 0.85 - 0.5 = 0.35$ m.

Hence, $k = \dfrac{F}{l - l_0} = \dfrac{19.6}{0.35} = 56$

(*b*)



We cannot carry on this graph beyond an extension of 1.50 m because after that point the spring is deformed.

(*c*) $k = 56$. Hence the Hooke's law for this spring gives

$F = 56x$

When a mass of 3.5 kg is applied this is equal to a force: $F = 9.8 \times 3.5 = 34.3$ N.

Then the extension is

$$x = \frac{34.3}{56} = 0.6125 = 0.61 \text{ m} \quad (2 \text{ s.f.})$$

(*d*) A mass of 10 kg corresponds to a force of 98 N. If the spring continued to stretch proportionally this would produce an extension of

$$x = \frac{98}{56} = 1.75 \text{ m}$$

However, this is beyond the *elastic limit* of the spring and we are **not** able to infer that the spring will stretch to this length. We are told that after 1.5 m the spring will deform. So we cannot determine the extension of the spring when this force is applied to it.

Remarks

(1) To say that Hooke's law is an empirical law of nature means that if we take springs and stretch them, we will discover that when a force is applied to a spring the extension of the spring is proportional to the force applied up to the elastic limit. Should you wish to confirm this law then you may acquire some springs and try stretching them using weights.

(2) As a further matter of fact, when springs are repeatedly stretched they suffer fatigue and at some stage become brittle. At this point the spring ceases to obey Hooke's law. You may try this yourself by repeatedly stretching elastic rubber bands. You will find that they become brittle after a while. Of course, in the questions here it is always assumed that the spring is "healthy" and obeys Hooke's law.

(3) Hooke's law applies to a very wide range of materials – metal wires, spring coils, elastic bands and so forth. You may assume that any question referring to elasticity or Hooke's law implies that the string, cord or spring obeys Hooke's law. This is conveyed in the question by the term *elastic*.

(4) Elastic springs under compression also obey Hooke's law.

Miscellany – we stand upon the shoulders of giants

Robert Hooke (1635 – 1703) was curator for experiments for the Royal Society at the time when Sir Isaac Newton was its President. Hooke criticised Newton's theory of light. (Newton believed that light was composed of particles, Hooke that it was a waveform. Nowadays both theories are simultaneously believed to be true in the theory of wave-particle duality.) The quarrel between them became very bitter. Hooke suffered from a congenital disease that made him effectively a hunchback. In a letter to Hooke (February 1676) Newton wrote, "If I have seen further it is by standing on the shoulders of giants", which may be taken as a sarcastic remark to effect that Hooke was not only physically deformed but an intellectual pigmy. The original metaphor of standing on the shoulders of giants is attributed by John of Salisbury (1159, *Metalogian*) to Bernard of Chartres. John wrote, "Beranrd of Chartres used to say that we are like dwarfs on the shoulders of giants, so that we can see more than they, and things at a greater distance, not by any virtue of any sharpness of sight on our part, or any physical distinction, but because we are carried high and raised up by their giant size." Despite Newton's sarcasm it is generally agreed that both men had brilliant minds, though, of course, Newton's stature as a genius cannot be surpassed by Hooke.

Young's modulus

The spring constant itself is only rarely given in questions. This is because it depends directly on another constant, called Young's modulus, which is of greater application in physics and engineering. The spring constant is given by

$$k = \frac{\lambda}{l_0}$$

where λ = Young's modulus, and l_0 is the natural length of the spring .

The units of Young's modulus

The units of the Young's modulus are newtons (N). We can see this from Hooke's law $F = kx$. Substituting $k = \frac{\lambda}{l_0}$ we get

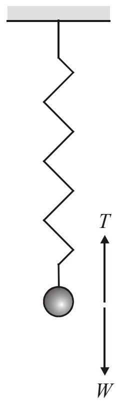
$$F = \frac{\lambda}{l_0}(l - l_0) = \lambda \frac{l_0}{l - l_0} .$$

The units of $\frac{l_0}{l - l_0}$ (metres) cancel out, leaving the units of F equal to those of λ. Hence the units of λ are newtons.

Example (3)

A light elastic spring with modulus of elasticity 160 N and of natural length 0.2 m hangs from a fixed point *O*. It has a particle of mass 4 kg attached to the other end. If the system is in equilibrium, hanging vertically from *O*, calculate the extension of the spring.

Solution



The forces are in equilibrium – i.e. the tension is equal to the weight.

$T = W$

Therefore

$kx = mg$

Here, $k = \frac{\lambda}{l_0} = \frac{160}{0.2} = 800$

$800x = 4 \times 9.8$

$x = \frac{4 \times 9.8}{800} = 0.049 \text{ m}$