Index Numbers

Converting a series into index numbers and calculations on series

In order to compare trends in data it is often easier to convert them into a simpler form.

It is usual when using indices to specify one value as equivalent to the index 100. Other values are then compared to this value.

These are calculations using ratios and are best illustrated by example.

Example

The following table shows figures for the United Kingdom Gross Domestic Product from 1991 to 2001 at constant 1995 prices. Convert them to index form taking the value for 1995 as equal to 100.[1]

Also, calculate the annual percentage change in GDP on a year by year basis.

Year	GDP / £m
1991	659,085
1992	651,566
1993	667,804
1994	698,915
1995	719,176
1996	738,046
1997	763,459
1998	785,777
1999	804,713
2000	829,517
2001	845,552

Solution

We use a ratio

The value of the year is to the value for 1995 as the unknown index number is to 100
The value for the year in 1991 was 659,985; therefore, for 1991 this gives

[1] Source: Office of National Statistics, *Blue Book*, 2002

$651,985 : 719,176 :: x : 100$

It will be clearer if this is written as a fraction

$$\frac{651,985}{719,176} = \frac{x}{100}$$

Hence

$$x = \frac{651,985}{719,176} \times 100 = 90.7\left(3.s.f.\right)$$

Year	GDP / £m	Index
1991	659,085	91.6
1992	651,566	90.1
1993	667,804	92.9
1994	698,915	97.2
1995	719,176	100
1996	738,046	102.6
1997	763,459	106.2
1998	785,777	109.3
1999	804,713	111.9
2000	829,517	115.3
2001	845,552	117.6

For the second part of the question, for accuracy, we use the original data, though we could calculate the year on year percentage changes using the index numbers as well.

The percentage change is the increase (or decrease) divided by the original value, expressed as a percentage (by multiplying by 100). This is again best shown by example.

For 1992 this is

$$\begin{aligned}\text{Percentage change} &= \frac{\text{Difference}}{\text{Original value}} \times 100 \\ &= \frac{\text{Value for 1992} - \text{Value for 1991}}{\text{Value for 1991}} \times 100 \\ &= \frac{651,556 - 659,985}{659,985} \times 100 \\ &= -1.14\%\end{aligned}$$

The negative value indicates that there was a recession in 1992.

Using this calculation we can complete the entire table

Year	GDP / £m	Index	% change
1991	659,085	90.7	
1992	651,566	90.1	-1.14
1993	667,804	92.9	+2.49
1994	698,915	97.2	+4.66
1995	719,176	100	+2.90
1996	738,046	102.6	+2.62
1997	763,459	106.2	+3.39
1998	785,777	109.3	+2.92
1999	804,713	111.9	+2.41
2000	804,713	115.3	+3.08
2001	845,552	117.6	+1.93

Weighted averages

An average of two numbers is found merely be adding the two numbers together and dividing by 2. Averages are also called in statistics the *mean* of a set of data. The average of the four numbers

6, 8, 10, 17

is found by adding them all together and dividing by 4; thus

$$\text{average} = \frac{6+8+10+17}{4} = \frac{41}{4} = 10.25$$

When we take a *weighted average* we count one number as being more "important" or "heavier" than another.

Example

Suppose a bag contains two kinds of ball bearing. One kind weighs 12 grams and the other 15 grams. Suppose 75% of the balls are of the first kind and 25% are of the second. What is the average weight of the balls in the bag?

Solution

Here we are being asked to find a weighted average of the ball bearings. We take the weights in the proportion to which the balls are in the bag. This is

$$\text{weighted average} = 75\% \text{ of } 12 + 25\% \text{ of } 15$$
$$= 0.75 \times 12 + 0.25 \times 15$$
$$= 12.75g$$

The retail price index

Inflation is a persistent rise in the general price level.

We know from experience that an object that costs, say £1 one year has a tendency to cost more in the next, and so on. This is the effect of inflation, which is to increase the price we pay in terms of the value of our currency on a year by year (or even, month by month or day by day) basis.

In order to keep track of inflation the government creates an index called the Retail Price Index (RPI). So inflation is a persistent increase in the Retail Price Index (RPI) measured on a year by year basis.

The Retail Price Index is a weighted average of the prices of goods and services – the index is weighted in the proportion to which an average household consumes these goods and services.

In order to calculate the retail price index the government first collects a huge range of prices from shops around the country. More than 100,000 individual prices are collected. However, a price index cannot be just the average of all these figures, because people spend more on one kind of item than another. For instance, the average household spends much more on heating fuel in a year than on paperclips. Consequently, the government takes a weighted average of these prices. The weights are chosen in accordance with the average expenditure of a household on each item in the list. Since there are thousands of prices in the list, we will illustrate the process with a simple example.

<u>Example</u>

Calculate the weighted average of prices for each of the following years and items. All figures are in £s.

Use the figures you calculate to determine a price index taking 1998 = 100.

Also calculate the percentage change in this price index on a year by year basis.

	Item			
	1kg of cheese	1 copy of *Pride and Prejudice*	1 litre of petrol	1 box of floppy discs
	Weight 40%	**Weight 10%**	**Weight 35%**	**Weight 15%**
1998	1.48	2.50	0.83	2.30
1999	1.55	2.50	0.88	2.15
2000	1.68	2.75	0.89	2.10
2001	1.78	3.00	0.95	1.99

The calculation for the first year (1998) is as follows

$$\begin{aligned}\text{weighted average} &= (0.4\times 1.48)+(0.1\times 2.5)+(0.35\times 0.83)+(0.15\times 2.30)\\ &= 1.4775\end{aligned}$$

Using this process we can fill in a column for the weighted average price

	Item				
	1kg of cheese	**1 copy of *Pride and Prejudice***	**1 litre of petrol**	**1 box of floppy discs**	
	Weight 40%	**Weight 10%**	**Weight 35%**	**Weight 15%**	**Weighted average price**
1998	1.48	2.50	0.83	2.30	1.4775
1999	1.55	2.50	0.88	2.15	1.5005
2000	1.68	2.75	0.89	2.10	1.5735
2001	1.78	3.00	0.95	1.99	1.6430

The index was found as before; for example, taking 1998 = 100, for 1999 this is

$$\frac{1.5005}{1.4775}=\frac{\text{index}}{100}$$

Hence

$$\text{index}=\frac{1.5005}{1.4775}\times 100=101.5$$

So a column for the index can be completed

	Item					
	1kg of cheese	**1 copy of *Pride and Prejudice***	**1 litre of petrol**	**1 box of floppy discs**		
	Weight 40%	**Weight 10%**	**Weight 35%**	**Weight 15%**	**Weighted average price**	**Index**
1998	1.48	2.50	0.83	2.30	1.4775	100
1999	1.55	2.50	0.88	2.15	1.5005	101.5
2000	1.68	2.75	0.89	2.10	1.5735	106.5
2001	1.78	3.00	0.95	1.99	1.6430	111.2

The percentage change in the average price (or index) is found as before. For example,

$$\begin{aligned}\text{Percentage change} &= \frac{\text{Difference}}{\text{Original value}}\times 100\\ &= \frac{\text{Value for 1999} - \text{Value for 1998}}{\text{Value for 1998}}\times 100\\ &= \frac{1.5005 - 1.4775}{1.4775}\times\\ &= 1.6\%\end{aligned}$$

	Weighted average price	**Index**	**% change**
1998	1.4775	100	
1999	1.5005	101.5	1.6
2000	1.5735	106.5	4.9
2001	1.6430	111.2	4.4

"Real" and "nominal" values

Because of inflation a country's national income, for instance, might *appear* to be growing when in fact it was not. All that was happening was that inflation was making the monetary values increase; but when inflation was taken into account there would be no *real* increase in the value of all the goods and services produces.

For example, if everything in the economy doubles in price, including all wages, but the total output of the country remains exactly the same, the monetary value of everything will have doubled but the real value will not have changed at all. Economists call this change in the *monetary value* of things the *nominal value*. In order to obtain a value for the *real* value they have to adjust the nominal value to take account of inflation.

This process is again best illustrated by example.

Example

The following table lists the nominal value of GDP for Britain from 1995 to 2001 at 2003 prices. It also gives the percentage change in the Retail Price Index (RPI) for each year. Find the real value of GDP at constant 1995 prices for each year.

Year	**% change in RPI**	**GDP at current prices / £m**
1995	3.3	719,176
1996	2.9	762,214
1997	2.8	811,067
1998	3.3	859,384
1999	2.4	902,459
2000	2.0	950,415
2001	2.7	988,014

Solution

In order to do this we have to recover the index and then deflate each figure on a year by year by the index.

Taking 1995 = 100, the index for 1996 is 102.9. To calculate the index for 1997 we have to find 2.8% of 102.9 and add that to 102.9, as follows

$$\text{Index} = 1.028 \times 102.9 = 105.8$$

Continuing in the same vein enables us to calculate the index column

Year	**% change in RPI**	**GDP at current prices / £m**	**RPI**
1995	3.3	719,176	100
1996	2.9	762,214	102.9

1997	2.8	811,067	105.8
1998	3.3	859,384	109.3
1999	2.4	902,459	111.9
2000	2.0	950,415	114.1
2001	2.7	988,014	117.2

Now we have to deflate each nominal GDP figure by the corresponding index. We illustrate this for 1996.

Using ratios

$$\frac{\text{Real value}}{\text{Nominal value}} = \frac{100}{102.9}$$

$$\begin{aligned}\text{Real value} &= \frac{100}{102.9} \times \text{Nominal value} \\ &= \frac{100}{102.9} \times 762,214 \\ &= 741,000 (3 s.f.)\end{aligned}$$

We can only quote to 3 s.f. because of that is the level of accuracy in the index numbers (strictly, we should quote only to 2 s.f.)

We can complete the table as follows

Year	**% change in RPI**	**GDP at current prices / £m**	**RPI**	**GDP at constant 1995 prices / £m**
1995	3.3	719,176	100	720,000
1996	2.9	762,214	102.9	741,000
1997	2.8	811,067	105.8	767,000
1998	3.3	859,384	109.3	786,261
1999	2.4	902,459	111.9	806,000
2000	2.0	950,415	114.1	833,000
2001	2.7	988,014	117.2	843,000

A formal definition of a *real* value is as follows: It is a measurement of an economic aggregate (such as Gross Domestic Product) corrected for changes in the price level over time, hence expressed in terms of constant prices for some base year.