Integrals of $\cos^2 x$ and $\sin^2 x$

To integrate $\cos^2 x$ and $\sin^2 x$ it is necessary to solve relevant trigonometric identities.

To integrate $\int \sin^2 x dx$

From the trigonometric identity we obtain a further identity for $\sin^2 x$:

$$\cos 2x = 1 - 2\sin^2 x$$

$$\therefore \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$$

$$\therefore \int \sin^2 x \, dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos 2x \right) dx$$

$$= \frac{x}{2} - \frac{1}{4}\sin 2x + c$$

To integrate $\int \cos^2 x dx$

$$\cos 2x = 2\cos^2 x - 1$$

$$\therefore \cos^2 x = \frac{1}{2} + \frac{\cos 2x}{2}$$

$$\therefore \int \cos^2 x \, dx = \int \left(\frac{1}{2} + \frac{1}{2}\cos 2x \right) dx$$

$$= \frac{x}{2} + \frac{1}{4}\sin 2x + c$$