# Linear Inequalities

## Prerequisites

You should already be aware that a linear expression in $x$ is a polynomial that has no term with a power higher than $x$. Thus $2x$ and $x-2$ are examples of linear expressions. You should have already met the symbols $>$ $\geq$ $<$ $\leq$ to express the following relationships.

$>$ means "greater than"
$\geq$ means "greater than or equal to"
$<$ means "less than"
$\leq$ means "less than or equal to"

The expression $3>-1$ is read, "3 is greater than –1" and the expression $1\leq 1$ is read, "1 is less than or equal to 1". Whereas a linear equation uses the equals sign to express an *exact* relationship between numbers, a *linear inequality* uses the symbols $>$ $\geq$ $<$ $\leq$ to express an *inexact* relationship. In the expression $x>-1$ the relationship is inexact because there are many numbers that satisfy this relationship, and this expression does not determine an exact number $x$.

## Solving linear inequalities

An example of a linear inequality is $2x+1>3x-2$. Linear inequalities are solved by the same algebraic manipulations required to solve a linear equation, except that when multiplying or dividing by –1 (or any negative number), then you must reverse the sign of the inequality.

**Example (1)**
Solve the linear inequality $2x>3x-2$.

Solution
$2x>3x-2$
$-x>-2$
$x<2$

In the last line of this example the sign of the inequality was reversed when both sides were multiplied by –1. This is really the only difference between the algebra of linear equalities and inequalities.

**Example (2)**

Solve the linear inequality $\frac{x+1}{4}-\frac{3x+1}{2}>\frac{x-1}{3}$.

Solution

$$\frac{x+1}{4}-\frac{3x+1}{2}>\frac{x-1}{3}$$
$$3(x+1)-6(3x+1)>4(x-1)$$
$$3x+3-18x-6>4x-4$$
$$-19x>-1$$
$$x<\frac{1}{19}$$

The solution to this problem differs from the solution to a linear equation occurs at the last stage when we divide both sides by $-19$, at which point we reverse the sign of the inequality.

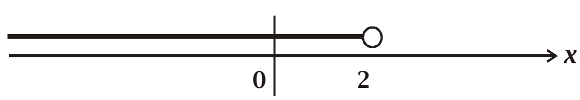
## Graphing linear inequalities (optional)

The solution to a linear equation can be presented as a segment of a straight line.

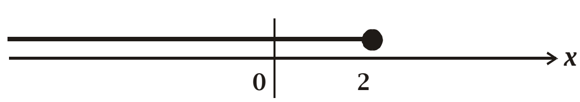
**Example (3)**

Sketch the graph of $x<2$ as a segment of a straight line.

Solution




This graph employs the following convention – a circle that has not been filled in represents a number that is not included in the solution set. Thus every number below 2 is a solution to the equation $x<2$ but 2 is not. To avoid confusion here the line representing the solution is drawn just above the line representing the set of all real numbers, positive and negative. The solution to $x\le 2$ would be represented by the following diagram.

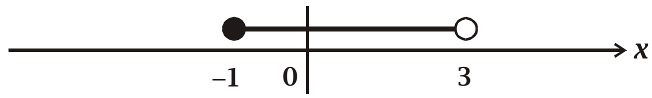



The difference here is that the circle has been filled in to indicate that the number 2 is a solution to $x\le 2$ as well as all the numbers below 2.

**Example (4)**

Sketch the set $-1 \le x < 3$

Solution




There is a second way of representing sets of numbers corresponding to linear equalities. This involves the use of brackets to represent the interval.

The set $x < 2$ is represented by $(-\infty, 2)$.

The set $x \le 2$ is represented by $(-\infty, 2]$.

The set $-1 \le x < 3$ is represented by $[-1, 3)$.

These employ the convention that a curved bracket is used when a point is *not* included in the solution set, and a square bracket is used when a point is included in the set. The symbols $-\infty$ and $+\infty$ stand for infinity and represent the idea that the line could be continued indefinitely. As infinity is not a number it cannot be included in a solution set, so a curved bracket always goes beside it. The symbol $(-\infty, +\infty)$ represents the whole real line. The symbols $\infty$ and $+\infty$ are interchangeable. The symbol $+\infty$ is used only when one needs to emphasise that the positive end of the real line is intended.