Linear momentum

What is momentum?

Imagine a train travelling along a track. How easy is it to bring that train to a halt? Experience shows that it will be more difficult to stop the train

(1) The faster the train is travelling

(2) The more massive the train is.

For example, a train with 15 carriages travelling at 100 km per hour will be far more difficult to bring to a halt than a train with 2 carriages travelling at 5 km per hour. In physics we give the name *momentum* to the physical property of moving objects that makes them more or less difficult to bring to a halt. We say that the fast and more massive train has more momentum than the slow and less massive train.

Definition of momentum

We define momentum to be

$\text{momentum} = \text{mass} \times \text{velocity}$

Since velocity is a vector, so is momentum. Momentum, like all vectors, has a direction. A train travelling due East has a different momentum to a train travelling due West. We shall use the symbol **p** to stand for momentum. So the definition of momentum in symbols is

$\mathbf{p} = mv$

Since $\text{momentum} = \text{mass} \times \text{velocity}$ the units of momentum are the units of mass $\times$ the units of velocity, which gives kgms^{-1}. However, the units of force are $\text{N} = \text{kgms}^{-2}$ and it is usual to write the units of momentum in terms of Newton seconds. The units are abbreviated to Ns.

Example (1)

What is the momentum of a toy train of mass 3 kg travelling along a straight, horizontal track with a velocity of 4.5 ms^{-1}?

Solution

$$\begin{aligned}\mathbf{p} &= mv \\ &= 3 \times 4.5 \\ &= 13.5 \text{ Ns}\end{aligned}$$

Impulse

In collisions momentum is transferred from one object to another. For example, in billiards, when the cue ball strikes a second ball all or part of the momentum of the cue ball is transferred to the second ball. The cue ball slows down and the second ball starts moving. When momentum is transferred this is called *impulse*.

impulse = change in momentum $\mathbf{J} = \Delta\mathbf{p}$

Here we use the symbol **J** for impulse and the Greek letter Δ to stand for "change in". Since impulse is transferred momentum it has the same units as momentum, Newton seconds (Ns).

Example (2)

A ball of mass 0.5 kg initially at rest is dropped from a height of 1.6 m onto the ground, which it strikes without bouncing.

(*a*) Find the velocity of impact.

(*b*) What is the impulse transferred by the ball to the ground at impact?

Solution

(*a*) To find the velocity this is a problem involving one of the equations of uniform acceleration. The ball is accelerating to the ground under gravity. The relevant equation is

$$v^2 = u^2 + 2as$$

with initial velocity $u = 0$, the acceleration due to gravity $g = 9.8$ and the distance $s = 1.6$. On substituting these values we obtain

$$\begin{aligned} v^2 &= 2 \times 9.8 \times 1.6 \\ &= 31.36 \end{aligned}$$

Hence

$$v = \sqrt{31.36} = 5.6\,\text{ms}^{-1}$$

(*b*) Since there is no bounce, the ball transfers all of its momentum to the ground. So the impulse is equal to the ball's momentum before impact.

$$\begin{aligned} \mathbf{J} &= \Delta\mathbf{p} \\ &= mv \\ &= 0.5 \times 5.6 \\ &= 2.8 \text{ Ns} \end{aligned}$$

In this solution we again use the units Newton seconds (Ns) rather than kgms^{-1}. Given the concept of impulse we can explain the relevance of this. Starting with the equation

$\mathbf{p} = mv$

Let us both multiply and divide the right-hand side by t, time, to get

$$\mathbf{p} = m \times \frac{v}{t} \times t$$

This is equivalent to multiplying by 1 so does not change the truth of the equation. Now $\frac{v}{t}$ is the definition of acceleration, so we have

$$\mathbf{p} = m \times a \times t$$

where a is acceleration. But Newton's second law is $F = ma$, which means that momentum and impulse are also given by

$$\mathbf{p} = Ft \qquad \text{momentum} = \text{force} \times \text{time}$$

This also enables us to look at impulse as the effect of a force F acting over a period of time t.

Example (3)

A footballer kicks a ball of mass 2 kg with a force of 30 N in a time of 0.2 s.

(*a*) What is the impulse imparted by the footballer to the ball?

(*b*) What is the velocity of the ball just after the footballer has kicked it?

Solution

(*a*) The impulse is given by

$$\Delta\mathbf{p} = Ft$$
$$= 30 \times 0.2 = 6 \text{ Ns}$$

(*b*) All of this impulse is transferred to the ball. Therefore the ball's momentum just after the footballer has kicked it is also 6 Ns. On substitution into $\mathbf{p} = mv$ we get

$$6 = 2 \times v$$
$$v = 3 \text{ ms}^{-1}$$

Linear momentum and conservation of momentum

At this stage we are only interested in momentum in one dimension – the kind of problems that we will be interested concern particles moving along a straight horizontal line or frictionless track. Momentum in one dimension is called *linear momentum.* In one dimension momentum can be in either a positive or a negative direction. You will no doubt be familiar with Newton's second law in the form of the equation

$$F = ma$$

In fact, Newton stated his law in terms of momentum. He stated that the rate of change of momentum of a *closed system* is zero. A *system* is a collection of particles that are physically related in some way. For example two balls connected by a light, inextensible string over a pulley constitute a system of connected particles. By a *closed* system we mean that no external forces are acting on any objects under consideration. So Newton's second law states that momentum cannot "leak out" of a closed system.

This law leads to the *principle of conservation of momentum*. This also says that the total momentum of any closed system is constant. In practice this means that if two particles collide, then the total momentum of the particles before the collision must be the same as the total momentum of the particles after the collision. Furthermore, if particle A loses momentum in the collision then the momentum particle A has lost is equal to the momentum that particle B has gained. We say that the impulse imparted by A to B is equal to the momentum gained by B

Principle of conservation of momentum for a collision between two particles

When two particles collide

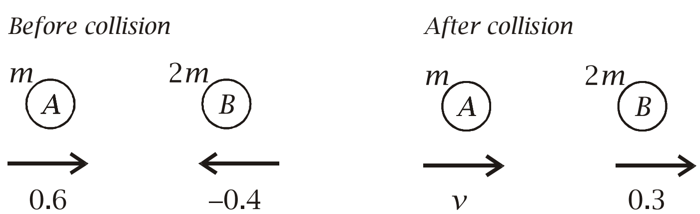
Total momentum before collision = Total momentum after collision.

Example (4)

Two particles A and B collide. Particle A has mass m and particle B has mass $2m$. They are moving towards one another on a smooth horizontal table. Immediately before impact A has speed 0.6 ms^{-1} and B has speed 0.4 ms^{-1}. Immediately after impact B has speed 0.3 ms^{-1} and is moving in the opposite direction to its motion before the impact. Find the speed of A immediately after impact.

Solution

Note, here "smooth" indicates as elsewhere, that friction can be ignored. The following diagram represents this situation



We need to recall that momentum and velocity are vectors. Therefore, if the positive direction is defined to be the direction in which particle A is moving, then the velocity of A is +0.6 ms^{-1} and the velocity of B before impact is –0.4 ms^{-1}. The total momentum before the impact is

$$\begin{aligned}\text{momentum before} &= (\text{momentum of } A + \text{momentum of } B)_{\text{before}} \\ &= (m_A v_A + m_B v_B)_{\text{before}} \\ &= m \times 0.6 - 2m \times 0.4 \\ &= -0.2m\end{aligned}$$

The momentum after impact is

$$\text{momentum after} = (\text{momentum of } A + \text{momentum of } B)_{\text{after}}$$
$$= (m_A v_A + m_B v_B)_{\text{after}}$$
$$= mv + 2m \times 0.3$$
$$= m(v + 0.6)$$

The total momentum is conserved.

momentum before = momentum after

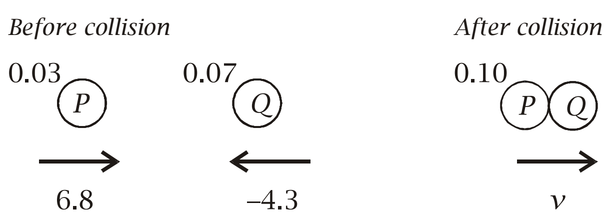
$$(m_A v_A + m_B v_B)_{\text{before}} = (m_A v_A + m_B v_B)_{\text{after}}$$
$$-0.2 = v + 0.6$$
$$v = -0.8 \text{ ms}^{-1}$$

This means that after impact A is moving with velocity $0.8\,ms^{-1}$ to the left. That is, in the opposite direction to that shown by the arrow in the diagram.

The annotation here makes the solution seem more complicated than it really is. In the next example we use minimal annotation.

Example (5)

Two particles P and Q are moving in a straight line towards one another when they collide and coalesce. Immediately before impact P is moving at 6.8 ms^{-1} and Q is moving at 4.3 ms^{-1}. The mass of P is 0.03 kg and the mass of Q is 0.07 kg. Find the speed and direction of the combined particle immediately after impact.



momentum before = momentum after

$$0.03 \times 6.8 - 0.07 \times 4.3 = 0.10 \times v$$
$$v = \frac{0.03 \times 6.8 - 0.07 \times 4.3}{0.10} = -0.97\,ms^{-1}$$

The speed is $0.97\,ms^{-1}$ and the combined particle is moving the direction in which Q was moving originally.