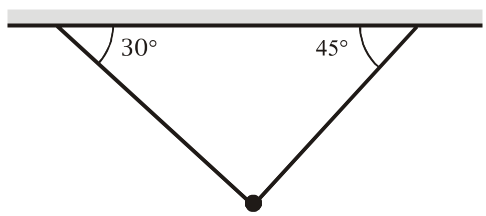
Normal Reaction and Friction

Prerequisites

You should be familiar with resolving forces and with the concept of static equilibrium. Let us revise these ideas.

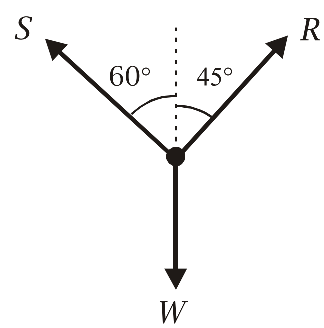
Example (1)

The diagram shows a particle of mass $40kg$ suspended from the ceiling by means of light inextensible strings inclined at angles 30° and 45° respectively. (*a*) Draw a force diagram and (*b*) find the tensions in the strings. Take $g = 9.8$.



Solution

(*a*) The force diagram is



Here the tensions in the strings are denoted by S and R respectively, and the weight of the particle by W.

(*b*) The weight is given by

$W = mg = 40 \times 9.8 = 392N$

Resolving horizontally and vertically

$(\rightarrow)$ $S\sin 60 = R\sin 45$

$$\frac{\sqrt{3}}{2}S = \frac{1}{\sqrt{2}}R \qquad (1)$$

$(\uparrow)$ $S\cos 60 + R\cos 45 = W$

$$\frac{1}{2}S + \frac{1}{\sqrt{2}}R = 392 \qquad (2)$$

Solving these two equations simultaneously

$$S = \frac{2}{\sqrt{2}\sqrt{3}}R$$

$$\frac{1}{2}\times\frac{2}{\sqrt{2}\sqrt{3}}R + \frac{1}{\sqrt{2}}R = 392$$

$$\frac{1}{\sqrt{3}}R + R = \sqrt{2}\times 392$$

$$R\left(1+\frac{1}{\sqrt{3}}\right) = \sqrt{2}\times 392$$

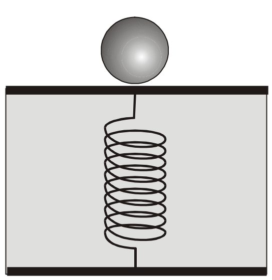
$$R = \frac{\sqrt{6}\times 392}{\left(1+\sqrt{3}\right)} = 351.457...N = 351N\ (3.s.f.)$$

$$S = \frac{2}{\sqrt{6}}\times 351.145... = 286.963... = 287N\ (3.s.f.)$$

In this example the particle is in *static equilibrium*. This means that the resultant force acting on the particle is zero and the particle is not accelerating.

Normal Reaction

Consider an object lying on the surface of a table. Let the object be in static equilibrium so its position relative to the surface does not change. The object “presses” down onto the surface with its own weight. Since the object is in static equilibrium the surface must be “pushing” back (reacting) with a force that is equal and opposite to the weight. This reaction is owing to the compression of the particles in the surface. The weight of the particle squeezes the molecules in the surface a little. The surface acts like a spring under compression and pushes back.



The surface acts like a spring under compression.

This force is called the *normal reaction* of the surface and is treated as acting perpendicularly to the surface. It is called the "normal force". The term "normal" is used because in mathematics it means "perpendicular to a surface or line".

Example (2)

A book of mass 3 kg rests on a surface. What is the magnitude of the normal reaction of the surface? ($g = 9.8$)

Solution

The book may be treated as a particle. The reaction is equal but opposite to its weight

$R = W = mg = 3 \times 9.8 = 29.4\,N$

Example (3)

A lift of mass 350 kg is going down at a constant velocity. What is the magnitude of the tension in the cable? ($g = 9.8$)

Solution

Despite the fact that the lift is moving, since it is not accelerating it is actually in static equilibrium. The tension in the cable must be equal to the weight of the lift, so

$T = W = mg = 350 \times 9.8 = 3430\,N$

Newton's Third Law

When an object pushes on a surface with a force, *F*, then the surface pushes on the object with a force, *G* that is in the opposite direction to *F* and is equal in magnitude to *F*. This is the idea of a normal reaction. However, this is just one application of a more general principle stated as *Newton's Third Law* of Motion: *Every action has an equal and opposite reaction.*

This law applies to all objects even if they are not in contact. If a body *A* exerts a force (or an action) on a body *B*, then *B* exerts an equal and opposite force (or reaction) on *A*. This expresses a fundamental principle of symmetry in the laws of nature. For example, if a body A exerts a gravitational force on a body *B*, then *B* exerts an equal and opposite gravitational force on *A*.

Example (4)

A hailstone of mass 0.1 kg plummets from the sky towards London. What is the force exerted by the hailstone on the Earth?

Solution

The force exerted by the Earth on the hailstone is its weight, which is

$W = mg = 0.1 \times 9.8 = 0.98\ N$

By Newton's Third Law the hailstone exerts an equal and opposite force on the Earth. That is, the Earth is "pulled" towards the hailstone with a force of 0.98 *N*.

Example (5)

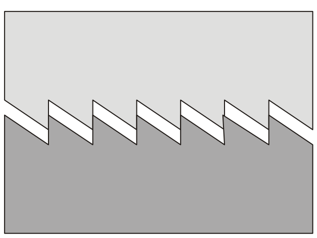
A man pushes against a wall with a force of 10 *N*. What is the force exerted by the wall on the man?

Solution

The wall exerts a force on the man that is equal and opposite to the force exerted by the man on the wall; this is 10 *N*.

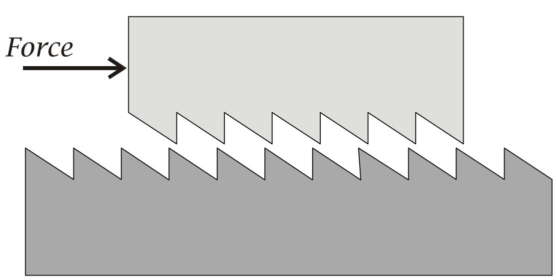
Friction

At the microscopic level surfaces are "rough". This means that when we try to pull one surface over another the rough parts catch against each other.

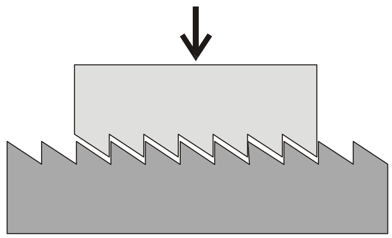


Contact between two surfaces produces friction.

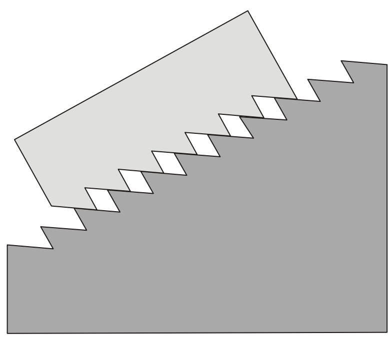
So the contact between the surfaces produces a resistance to movement that is called *friction*. The friction acts along the surface and opposes the direction in which the object would move if there were no friction. However, if you push an object along a surface with sufficient force then you can overcome the frictional resistance and the object will "slide" along the surface.



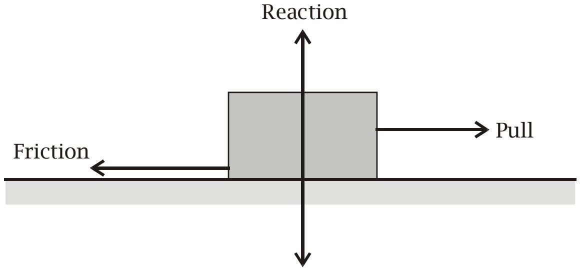
A heavy object presses down upon the surface it rests on more than a light one.



So the weight of an object increases the contact between the rough surfaces – the friction increases with the weight of the object lying on a surface. However, if a surface is inclined at an angle



then the amount of contact between an object resting on it and the surface is reduced; therefore, so too is the friction. Experiment shows that that the size of the frictional force depends on the magnitude of the normal reaction at a surface. The following diagram shows an object resting on a level surface. Someone is trying to pull it along the surface and this pull is being resisted by friction.



As the force applied to the object along a surface is increased, eventually the frictional resistance to motion is overcome, and the object begins to move. The point at which the object is about to move is called *limiting equilibrium*. The frictional force at limiting equilibrium is a fixed fraction of the normal reaction. Once limiting equilibrium has been overcome the frictional force does not increase. Thus, at limiting equilibrium

$F = \mu R$

where μ is a constant called the *coefficient of friction*. Since the frictional force cannot be greater than the force that is trying to push (or pull) an object, below limiting equilibrium

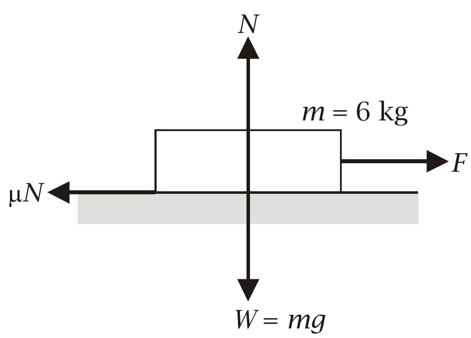
$F < \mu R$.

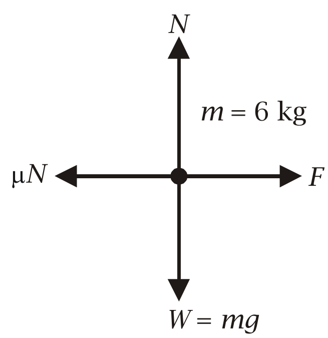
The frictional force is just sufficient to prevent the object sliding along the surface.

Example (6)

A mass of 6kg lies on a rough table where the coefficient of friction μ is 0.45. Find the least horizontal force needed to move it. $g = 9.8$

Solution





We model this object as a particle; the diagram above shows the forces acting on it.

We are given that $\mu = 0.45$

The normal reaction is equal to the weight and is given by

$N = 6g$

At the moment that the object is about to move we have limiting friction, so

$F = \mu N$

Hence

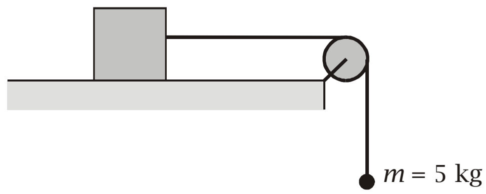
$F = 6\mu g = 6 \times 0.45 \times 9.8 = 26.5\text{N} \; (3 \text{ s.f.})$

Smooth surfaces

Questions in mechanics frequently refer to the term *smooth*. A *smooth* surface is one that offers no frictional resistance – so the coefficient of resistance μ is zero. In fact, in the real world, such surfaces do not exist, but air-tracks and ice-rinks are nearly smooth.

Example (7)

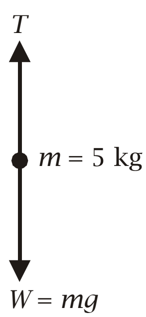
The diagram shows a mass of 5 kg suspended from the edge of a table by means of a *light inextensible* string hung over a *smooth* pulley and attached to a block resting on the top of the table. Given that the entire system is in static equilibrium find the tension in the string and the frictional force exerted by the table on the block.



Note. In this question we also meet the terms *light* and *inextensible*. The term *light* means that the object (in this case, the string) has no weight. This is obviously not true, but it is a convenient way of saying that in this question the weight of the string can be ignored. The term *inextensible* means that the string cannot stretch. This means that the tension in the string can be treated as constant and that the whole system is not, for instance, oscillating – bobbing up and down. These terms are just convenient ways of saying that for the purpose of the problem no further ideas other than the ones already given need to be considered.

Solution

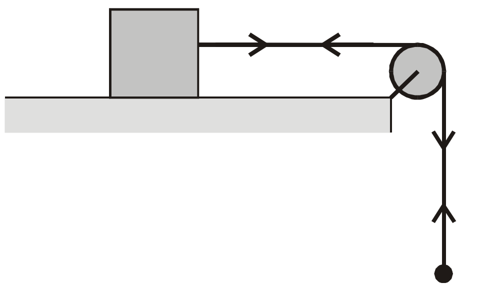
Since the pulley is smooth there is no reaction or friction to consider at the pulley. So the only force in the string to consider is the tension produced by the weight of the 5 kg mass. Resolving the forces at the 5 kg mass we have the following force diagram.



The diagram makes it clear that the tension in the string is equal to the weight of the 5 k mass. That is

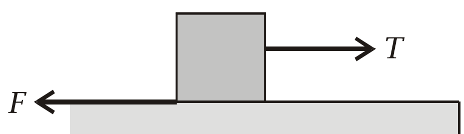
$T = W = mg = 5 \times 9.8 = 49\,N$.

Note. Before we continue with the solution, we should add a note about cables. In this example the cable is not moving, that is, it is in static equilibrium. There is a tension in the cable pulling the 5 kg mass up, but equally there is a tension in the cable pulling the cable down. At all times the two tensions cancel each other. This is another application of Newton's Third Law – the *action* of the tension in one direction is equal and opposite to the *reaction* of the tension in the opposite direction. Where necessary we represent this idea on a diagram by showing arrows pointing in *both directions.*



Solution continued

The note above makes it clear that there is a tension in the string that is "pulling" the block along the surface. However, as the block is not moving, the friction at the table surface must be equal and opposite to this tension



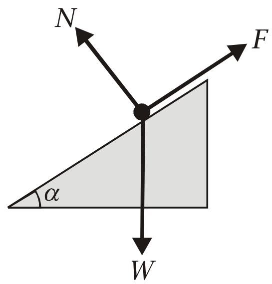
Hence the friction is

$F = T = 49\ N$

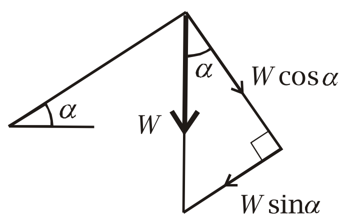
Note also that in this question we were not told anything about the coefficient of friction at the surface of the table. The block is either at or below the limit of static equilibrium. The coefficient of friction or the mass of the block is not required in this question.

Slopes and motion down slopes

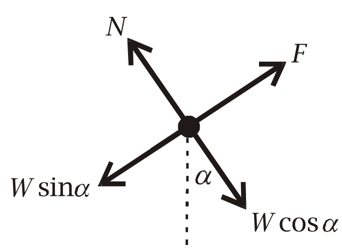
An object that is at rest on a slope is subject to three forces: its weight pulling it down the slope; a normal reaction preventing it from falling through the surface; a frictional force preventing it form sliding down.



The normal reaction acts perpendicularly to the sloping surface and is equal to the component of the weight that is also perpendicular to the surface. The force that could cause the body to slide down the slope is the component of the weight acting down the slope. These components are $W\cos\alpha$ and $W\sin\alpha$ respectively.



Note how the angle of the slope α is equal to the angle between the direction of the weight (*W*) and the component of the weight that is perpendicular to the surface of the slope $(W\cos\alpha)$. This follows from elementary geometry of the triangle.



So resolving parallel and perpendicular to the slope we find

$(\nwarrow)$ $N = W\cos\alpha$ $\qquad$ $(\nearrow)$ $F = W\sin\alpha$

Here the friction is just sufficient to prevent the object from moving, so it is determined by the component of the weight acting down the surface of the slope. However, if the system is at limiting equilibrium then the friction is given by

$F = \mu N$

where N is the normal reaction, and we have

$F = \mu N = W \sin\alpha$

Example (8)

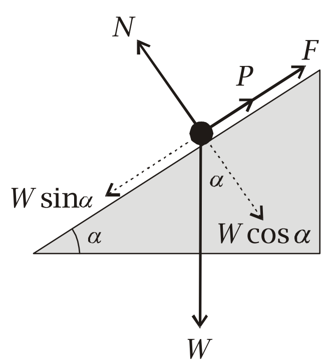
A body of mass 4 kg lies on a slope of angle 30°, where the coefficient of friction is 0.3.

(a) Find the least force parallel to the slope that will prevent it from siding down.

(b) Find the least force parallel to the slope that will push it up the slope.

Solution

(a) Let P be the force preventing the object from sliding down the slope. In this case the friction assists this force.



$m = 4kg$

$\alpha = 30°$

$\mu = 0.3$

Resolving perpendicularly to the slope

$(\nwarrow)$ $N = W\cos\alpha$

$= mg\cos\alpha = 4 \times 9.8 \times \cos 30° = 33.9481...$

Resolving parallel to the slope

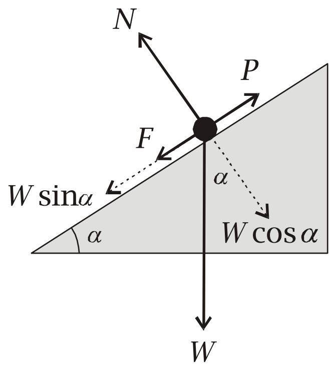
$(\nearrow)$ $P + F = W\sin\alpha$

$P = W\sin\alpha - F$

$P = mg\sin\alpha - \mu N$

$P = 4 \times 9.8 \times \sin 30° - 0.3 \times 33.9481... = 9.42\,N\ (3\ s.f.)$

(b) Let P be the force preventing the object from sliding down the slope. In his case the friction resists this force.



Resolving perpendicularly to the slope

$(\nwarrow)\quad N = W\cos\alpha$

$\quad = mg\cos\alpha = 4\times 9.8\times\cos 30° = 33.9481...$

Resolving parallel to the slope

$(\nearrow)\quad P = W\sin\theta + F$

$P = mg\sin\theta + \mu N$

$P = 4\times 9.8\times\sin 30° + 0.3\times 33.9481... = 29.8N\,(3.S.F.)$

Reaction at the floor of a lift

When an object is placed in a lift the floor of the lift must not only support the weight of that object, but also supply the force that causes the crate to move upwards. If the lift is descending then the reaction at the floor of the lift will be less than the weight of the crate.

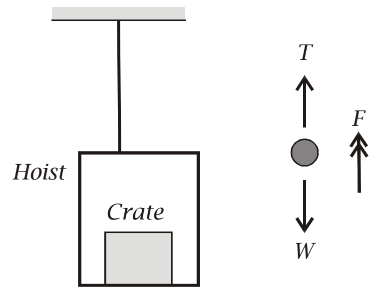
Example (9)
A crate of mass 300 kg is placed into a hoist of mass 120 kg attached to a cable. Find

(*a*) The tension in the cable when the lift is accelerating upwards at 2.8 ms^{-1}.

(*b*) The magnitude of the reaction of the floor of the hoist on the crate.

Solution

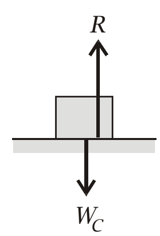
(*a*) In the first part of this question we treat the hoist and the life together as a single particle of combined mass 300 + 120 = 420 kg subject to a tension in the cable and its combined weight. The lift is accelerating upwards so the tension in the cable is greater than its weight, and the resultant force produces the acceleration.



Then

$F = T - W$
$ma = T - mg$
$420 \times 2.8 = T - 420 \times 9.8$
$T = 1176 + 4116 = 5292$ N

(*b*) Now we look at the problem from the crate's point of view. The crate is subject to two forces, the normal reaction at the floor *R* and its weight W_C. The normal reaction is greater than the weight and the resultant produces acceleration also of 2.8 ms^{-1}.



$F = R - W_C$
$m_C a = R - m_C g$
$120 \times 2.8 = R - 120 \times 9.8$
$R = 1512$ N