Paired-sample sign test on the equality of population medians

This is a non-parametric test based on ordinal level data. The source of the data are a set of paired values from two samples. The hypothesis is that the population median of both sets of data are the same. There does exist a non-parametric test that uses the fact that the data can be ranked. It is called the paired-sample Wilcoxon signed rank test on equality of population medians. However, the paired-sample sign test is a "simpler" test that first reduces the data to nominal level.

Example

In the past suspects of drink driving were required to walk on a white line. Ten volunteers were asked to walk a white line before and after drinking two pints of a Manchester ale. The distances travelled before wobbling off were measured to the nearest metre, and the following results were obtained.

Volunteer	1	2	3	4	5	6	7	8	9	10
Before ale	10	9	11	4	8	12	6	7	8	8
After ale	8	6	11	5	6	13	3	1	9	4

Test at the 5% significance level whether getting drunk on Manchester ale makes a difference to your ability to walk straight.

H_0: there is no difference in the medians.
H_1 median value after drinking ale is less than median value before drinking ale.

We proceed by considering the sign of the difference.

Volunteer	1	2	3	4	5	6	7	8	9	10
Sign of difference	+	+	0	-	+	-	+	+	-	+

Let X = number of negative signs.
We delete the 0 score from the test.

Then $X \sim B(9, 0.5)$

$$P(X \le 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$
$$= \left({}^{9}C_{0} + {}^{9}C_{1} + {}^{9}C_{2} + {}^{9}C_{3}\right)(0.5)^{9}$$
$$= (1 + 9 + 36 + 84)(0.5)^{9}$$
$$= 0.2539$$

$$\therefore P(X \le 3) \ > \ 0.05$$

Hence, we accept H_0 and reject H_1

Apparently this beer has not made any difference to the performance of the white-line artistes.