Paired sample Wilcoxon signed rank test on equality of population medians

This is a non-parametric test concerning a set of paired values from two samples. We are testing the hypothesis that the population median of both sets of data are the same. In the paired-sample sign test we reduce the level to nominal level. But as the data is in fact ordinal we can rank the differences of the two samples. This enables us to construct a test statistic that can be compared to a critical value drawn from Wilcoxon's T distribution.

To illustrate this we reconsider the example about beer drinking and white lines introduced when we met the paired-sample sign test.

Example

In the past suspects of drink driving were required to walk on a white line. Ten volunteers were asked to walk a white line before and after drinking two pints of a Manchester ale. The distances travelled before wobbling off were measured to the nearest metre, and the following results were obtained.

Volunteer	1	2	3	4	5	6	7	8	9	10
Before ale	10	9	11	4	8	12	6	7	8	8
After ale	8	6	11	5	6	13	3	1	9	4

Use the paired sample Wilcoxon signed rank test to test at the 5% significance level the claim that drinking beer reduces your ability to walk on little white lines.

H_0: there is no difference in the medians.
H_1: median value after drinking is less than median value before drinking.

This is a one-tailed test.

We begin by calculating the difference between the sample values and ranking those differences in order of magnitude.

Volunteer	Before	After	Difference D	Rank of D	Signed Rank
1	10	8	2	4.5	+4.5
2	9	6	3	6.5	+6.5
3	11	11	0	--	--
4	4	5	-1	2	-2
5	8	6	2	4.5	+4.5
6	12	13	-1	2	-2
7	6	3	3	6.5	+6.5
8	7	1	6	9	+9
9	8	9	-1	2	-2
10	8	4	4	8	+8

The table illustrates that, as usual, zero values are deleted from the set of data. When ranks are tied the rank is the average of the tied ranks.

Let T_+ denote the sum of the ranks of the +ve value.
Let T_- denote the sum of the ranks of the –ve value.

The test statistic, denoted by T, is the smaller of these two values.

Here $T_+=39$ $T_-=6$ $T=T_-=6$

$T_{critical} = 8$ (p=0.05, n=9, one-tailed)

$T_{test} < T_{critical}$

$\therefore$ reject H_0, accept H_1

Thus, the white line test does reveal that drunkenness impairs performance after all!