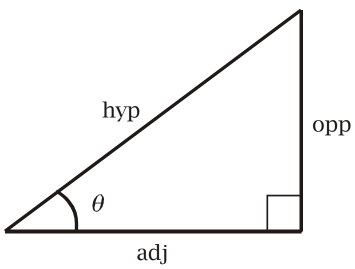
Plane Trigonometry

Prerequisites

You should already be familiar with trigonometry, and have met the expressions $\sin x$, $\cos x$ and $\tan x$. You should be familiar with the use of trigonometry to find angles, bearings, sides and distances in geometrical figures or problems.

The following diagram represents a right-angled triangle.



The side next to the angle *x* is called the *adjacent* side, abbreviated to *adj*; the side opposite *x* is called the *opposite* side (*opp*); and the longest side that is opposite the right-angle is called the *hypotenuse* (*hyp*). The trigonometric ratios are

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

where sin, cos and tan are abbreviations for sine, cosine and tangent respectively. Most people develop a device for remembering these ratios. One useful way is to remember the 'nonsense' word

SOHCAHTOA

By quickly scribbling this down, you can recall the ratios

$$\text{SOH} \longrightarrow S = \frac{O}{H} \longrightarrow \sin\theta = \frac{opp}{hyp}$$

$$\text{CAH} \longrightarrow C = \frac{A}{H} \longrightarrow \cos\theta = \frac{adj}{hyp}$$

$$\text{TOA} \longrightarrow T = \frac{O}{A} \longrightarrow \tan\theta = \frac{opp}{adj}$$

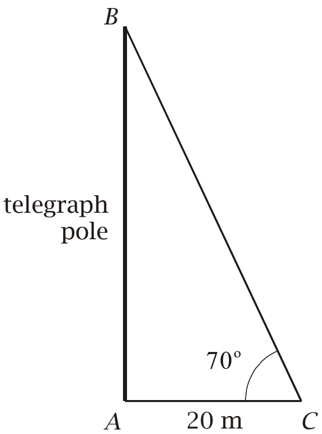
Problem solving using trig

Knowledge of the trigonometric ratios is used to solve certain problems.

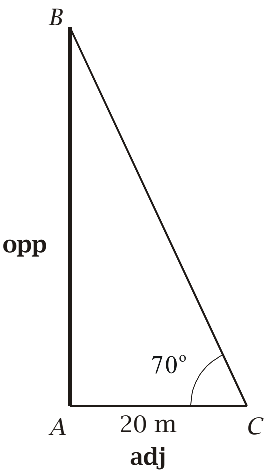
Example (1)

A telegraph pole casts a shadow 20m long when the altitude of the sun is 70°. What is the height of the flagpole? Give your answer to 3 significant figures.

Solution



Here the line AB represents the telegraph pole, and the line AC the shadow cast by the pole. The angle between the end of the shadow and the top of the pole is 70°. The height of the pole is opposite the angle we know, the length of the shadow is adjacent to it, so we choose tan.



$$\tan 70 = \frac{AB}{AC}$$

On substituting $AC = 20$

$$\tan 70 = \frac{AB}{20}$$

By rearrangement

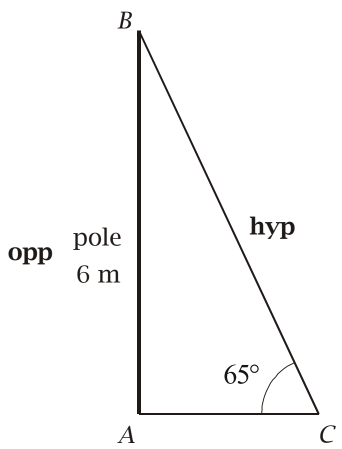
$$AB = 20 \tan 70$$

Evaluating

$$\begin{aligned} AB &= 20 \times 2.7474... \\ &= 54.9495... \\ &= 54.9 \text{ m} \left(3\text{.s.f.}\right) \end{aligned}$$

Example (2)

A man is erecting a marquee. He has a central pole that is 6 metres high. He needs to anchor this to the ground by guy-ropes that will make an angle of 65° with the ground. What will be the length of each guy-rope? Give your answer to 3 significant figures.



$$\sin 65 = \frac{AB}{BC}$$

On substituting the length of the pole, we get

$$\sin 65 = \frac{6}{BC}$$

Rearranging

$$\begin{aligned} BC &= \frac{6}{\sin 65} \\ &= \frac{6}{0.9063...} = 6.62926... = 6.62 \text{ m } \left(3\text{.s.f.}\right) \end{aligned}$$

Taking the inverse

A final type of problem concerns the finding of an angle, given two sides of a triangle. To do this, you need to be familiar with the process of taking the inverse of sine, cosine or tangent. The trigonometric ratios are processes (we call them *functions*) that take us from an angle to the value of a ratio.

$$\left.\begin{matrix}\theta \\ \text{angle}\end{matrix}\right\} \xrightarrow{\sin} \left\{\begin{matrix}\sin\theta \\ \text{ratio} = \dfrac{opp}{hyp}\end{matrix}\right.$$

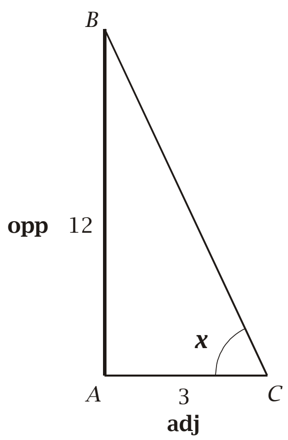
We can *reverse* this process, and work backwards from the ratio to the angle.

$$\sin\theta \xrightarrow{\sin^{-1}} \theta$$

This is called the *inverse*. The inverse of the function $\sin\theta$ is written $\sin^{-1}\theta$ which is pronounced "inverse sine". (In some textbooks it is written $\arcsin\theta$ which is pronounced "arc-sine".)

Example (3)

A flagpole is 12 metres high. It is fastened by ropes to the ground so that the point of attachment is 3 metres from the base of the pole. What is the angle made by the ropes with the ground? Give your answer to 0.1°.



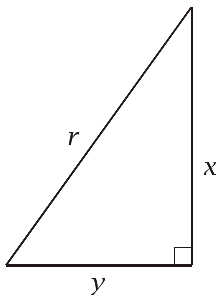
$$\tan x = \frac{opp}{adj} = \frac{12}{3} = 4$$

$$x = \tan^{-1} 4$$

$$\begin{aligned} x &= 75.96...^\circ \\ &= 76.0^\circ \; (0.1^\circ) \end{aligned}$$

Relationships between the ratios

When one ratio is given, the other ratios can be calculated without recourse to finding the angles of the triangle. *Pythagoras's theorem* states that *in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares of the other two sides.*



So we have

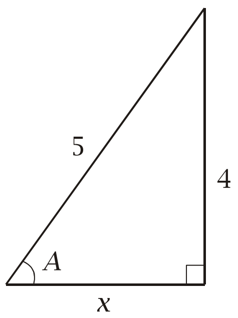
$r^2 = x^2 + y^2$

Example (4)

In a right-angled triangle, if $\sin A = \frac{4}{5}$, find $\tan A$ and $\cos A$.

Solution

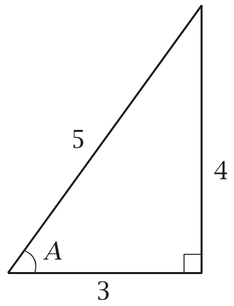
We have a right-angled triangle, and we are given $\sin A = \frac{4}{5}$. This means that the ratio of the side opposite to the angle A to the hypotenuse is 4 to 5.



Here x denotes the unknown side in the triangle. By Pythagoras's theorem.

$5^2 = x^2 + 4^2$
$25 = x^2 + 16$
$x^2 = 25 - 16$
$x^2 = 9$
$x = 3$

Hence the triangle has sides 3, 4 and 5.



The other ratios are given by the triangle as

$\tan A = \frac{4}{3} \qquad \cos A = \frac{3}{5}$

The numbers 3, 4 and 5 are known as a *Pythagorean triple.* They are whole numbers that satisfy the relationship

$5^2 = 4^2 + 3^2$

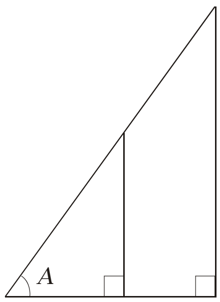
Any three whole numbers, x, y and z such that

$z^2 = x^2 + y^2$

are called a Pythagorean triple. There are an infinite number of these. The numbers 5, 12 and 13 are another example.

Trigonometric ratios are just ratios

The value of a trigonometric ratio does not depend on the size of the triangle. It is a ratio. That is why in the example (4) it was unnecessary to consider whether the triangle had a side of, say, length 5 metres. The actual lengths are not needed. In a ratio the units of measurement cancel out. The following triangle illustrates this idea.



The sine, cosine and tangent of the angle A are the same, no matter what the size of the triangle.