Power

Prerequisites

You should be familiar with (1) Newton's second law and with resolving forces, including problems concerning the static equilibrium of an object resting on an inclined surface; (2) the definitions of work and energy.

Example (1)

A car, of mass 1000 kg, is travelling up a hill inclined at an angle α where $\sin\alpha = 0.1$ at a constant speed of $6\ \mathrm{ms}^{-1}$. There is a constant resistance to the motion of the car of $1600\ \mathrm{N}$.

(*a*) Find the force exerted by the engine of the car.

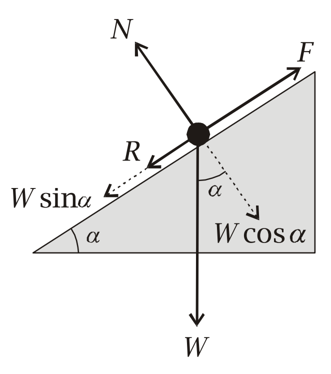
(*b*) If the car travels 1200 m up the hill find the work done by the engine of the car.

(*c*) Find how long it takes for the car to travel this distance of 1200 m.

(*d*) Find the rate at which the car's engine is working expressing your answer in kJ per second.

Solution

The following diagram shows the forces acting on the car.



These are (1) the force exerted by the car's engine (F), (2) the resistance to the motion of the car from the surface and the air, which is here $R = 1600\ \mathrm{N}$, (3) the weight of the car,

$W = mg = 1000 \times 9.8 = 9800$ N and (4) the normal reaction (N). The weight is resolved into components acting along the slope and perpendicular to the slope.

(*a*) Since the car's velocity is constant there is no resultant force acting on the car, so the forces must be in equilibrium. Resolving parallel to the line of the slope

$$(\nearrow) \quad F = R + W \sin\alpha$$
$$= 1600 + 9800 \times 0.1$$
$$= 2580 \text{ N}$$

(*b*) work = force × distance

$$= 2580 \times 1200$$
$$= 3096000$$
$$= 3100 \text{ kJ (3 s.f.)}$$

(*c*) $v = \dfrac{d}{t}$ $\qquad \left[\text{velocity} = \dfrac{\text{distance}}{\text{time}}\right]$

$$t = \frac{d}{v} = \frac{1200}{6} = 200 \text{ s}$$

(*d*) rate of working $= \dfrac{\text{work}}{\text{time}} = \dfrac{3096}{200} = 15.5 \text{ kJs}^{-1}$ (3 s.f.)

A car's engine is an example of a machine. Every machine is a device for converting one form of energy to another. This is the work that the machine does. A car's engine converts the chemical potential energy of the car's fuel into the mechanical energy of the turning of the car's wheels. When any machine converts energy it works at a certain rate. We call this rate of working (the rate at which energy is converted) *power*. Power is therefore defined as

$$\text{Power} = \text{rate of conversion of energy} = \frac{\text{Change in energy}}{\text{Change in time}} \qquad P = \frac{E}{t}$$

The rate of working of an engine or machine is the same as the rate at which it converts one form of energy to another; both are power.

Remark

No machine or engine is (or can be) 100% efficient. For example, a car's engine is designed to convert the chemical potential energy of its fuel into mechanical energy. However, in the process the car's engine also gets hot, so other useless and undesired energy conversions also take place. In this chapter we ignore the loss of energy as heat and are only interested in the rate at which useful energy is converted.

The units of power

As the answer to part (*d*) of example (1) above indicates power may be expressed in terms of energy converted per second, that is joules per second, Js^{-1}. However, just as the units of energy (joules, J) are a shorthand for

$\text{units of energy} = \text{kg} \times \text{ms}^{-2} \times \text{m} = \text{kgm}^2\text{s}^{-2}$

so we use watts, symbol W, for the units of power. 1 watt is a rate of working (energy conversion) of 1 joule per second.

The power force relationship

Since

$$\text{Power} = \text{rate of conversion of energy} = \frac{\text{Change in energy}}{\text{Change in time}} \qquad P = \frac{E}{t}$$

We can rearrange this equation to obtain

$\text{energy} = \text{power} \times \text{time}$

$E = P \times t$

Since energy = force × distance, this means

$$P = \frac{E}{t} = \frac{Fd}{t} \qquad (1)$$

Furthermore, velocity is defined as the ratio of distance to time.

$$v = \frac{d}{t}$$

Hence, on substituting this into (1)

$$\text{power} = \text{force} \times \text{velocity} \qquad P = Fv$$

This provides a relationship between power, force and velocity. We shall call it the *power force relationship*.

Example (2)

An athlete pulls a trolley at a velocity of $2.5\ \text{ms}^{-1}$ against a resistance of 50 N. Find the power of the athlete

Solution.

$\text{Power} = Fv = 50 \times 2.5 = 125\text{W}$

Energy is a scalar quantity and so too is power. However, it is customary to write the power force relationship in terms of velocity rather than speed.

$$\text{power} = \text{force} \times \text{velocity} \qquad P = Fv$$

However, in this equation it is really the magnitude of the velocity that is being considered – that is the speed. In practice no difficulties arise from this.

The power force relationship enables us to determine the force exerted by an engine, given its power and speed.

Example (3)

A car is moving along a straight level road. The power of the engine is 20 kW. The total mass of car and driver is 500 kg. Find the acceleration when the speed is 15 ms^{-1}. Ignore air-resistance.

Solution

$$P = Fv$$

Substituting $P = 20{,}000$ and $v = 15$

$$F = \frac{P}{v} = \frac{20000}{15} = 1333...N$$

$$F = ma$$

$$a = \frac{F}{m} = \frac{1333}{500}$$

$$= 2.66...$$

$$= 2.7\text{ms}^{-2} \quad (2 \text{ s.f.})$$

Example (4)

A car of mass 1200 kg is pulling a trailer of mass 600 kg up a hill inclined at an angle α at a constant speed of 10 ms^{-1}. The car's engine is working at a rate of 35 kW. The resistance to motion acting on the car is 200 N and that acting on the trailer is 150 N.

(*a*) Find the value of α to 0.1 degree.

(*b*) The driver of the car suddenly increases the power of the car to 50 kW. Find the acceleration of the car and trailer at the instant he does this. Find the tension in the rigid tow-bar connecting the car to the trailer at this moment.

(*c*) What will be the new maximum velocity of the car and trailer when the car is working at 50 kW?

Solution

(*a*) The force exerted by the engine of the car is

$$F = \frac{P}{v} = \frac{35000}{10} = 3500 \text{ N}$$

The total resistance acting on the car and trailer is 350 N.

The total mass of the car and trailer is 1800 kg.

The component of the weight of the car and trailer combined that acts down the slope is $W \sin\alpha = mg\sin\alpha = 1800 \times 9.8 \times \sin\alpha = 17640\sin\alpha$.

The car is travelling at a constant speed so the forces are in equilibrium.

Resolving parallel to the slope.

$$(\nearrow) \qquad F = R + W\sin\alpha$$
$$3500 = 350 + 17640\sin\alpha$$
$$\sin\alpha = \frac{3150}{17640} = 0.178...$$
$$\alpha = 10.3^\circ \ (0.1^\circ)$$

(*b*) The force of the engine increases to

$$F = \frac{P}{v} = \frac{50000}{10} = 5000 \text{ N}$$

There is now a resultant force acting on the car and trailer, which is

$$\begin{aligned}\text{resultant} &= F - R - W\sin\alpha \\ &= 5000 - 350 - 3150 \\ &= 1500 \text{ N}\end{aligned}$$

So the acceleration of the car and the trailer is

$$a = \frac{\text{resultant}}{\text{mass of car and trailer}} = \frac{1500}{1800} = \frac{5}{6} \text{ ms}^{-2}$$

The forces acting on the trailer parallel to the slope are the tension in the tow-bar the component of its weight and the resistance to motion. The tension is greater than the weight and resistance combined and there is a resultant force that accelerates the trailer at $\frac{5}{6}$ ms^{-2}. Thus the resultant is

$$\text{resultant} = ma = 600 \times \frac{5}{6} = 500 \text{ N}$$

Then

$$\text{resultant} = \text{tension} - \text{component of weight} - \text{resistance}$$
$$500 = T - mg\sin\alpha - 150$$
$$T = 500 + 600 \times 9.8 \times \left(\frac{3150}{17640}\right) + 150 = 1700 \text{ N} \quad (3 \text{ s.f.})$$

(*c*) At the maximum velocity the resultant force is zero. Then the force of the engine is again 3500 N.

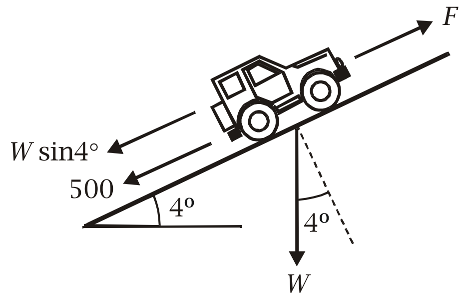
$$P = Fv \quad \Rightarrow v = \frac{P}{F} = \frac{50000}{3500} = 14\frac{2}{7} \text{ ms}^{-1}$$

In the following example we use energy considerations to solve a problem.

Example (5)

A car is of mass 650 kg moving up a straight hill, inclined at an angle of 4º to the horizontal. Its engine has a power of 50 kW. Initially it is at rest and it takes 18.1 s to reach 80 kmh^{-1}. The resistance to the motion of the car is constant at 500 N. Using energy considerations find the distance travelled during this time.

Solution



$80\,\text{kmh}^{-1} = \dfrac{80 \times 1000}{60 \times 60} = 22.2\ \text{ms}^{-1}$

Since $P = \dfrac{E}{t}$

Work done by engine = $P \times t$

By the work-energy principle

work done by engine	=	kinetic energy generated	+	work done against resistance	+	gain in gravitational potential energy

$$Pt = \frac{1}{2}mv^2 + Fd + mg\sin\theta d$$

$$50000 \times 18.1 = 0.5 \times 650 \times (22.2)^2 + 500 \times d + 650 \times 9.8 \times \sin 4 \times d$$

$$d = \frac{50000 \times 18.1 - 0.5 \times 650 \times (22.2)^2}{(500 + 650 \times 9.8 \times \sin 4)} = 788.38... = 790\text{m} \quad (2\text{ s.f.})$$