Projectiles

Prerequisites

You should be familiar with the equations of uniform acceleration.

Example (1)
A particle is moving along a straight horizontal smooth track. It passes a point P at time $t_0 = 0$ with velocity 6 ms^{-1}. At time t_1 it passes the point Q with velocity 2 ms^{-1}. Given the distance between P and Q is 8 m, find the acceleration of the particle.

Solution
The problem may be recast as, given $s = 8, u = 6$ and $v = 2$, find a. To solve this we substitute into the equation

$$v^2 = u^2 + 2as$$

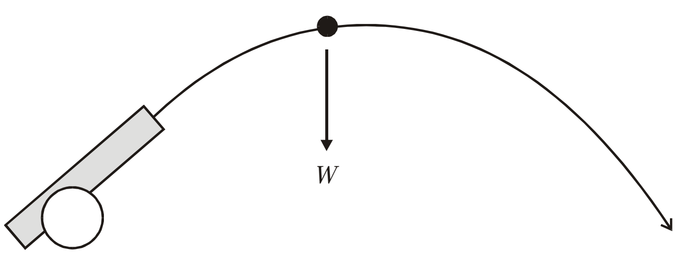
where $s = 8 \quad u = 6 \quad v = 2$

$$(2)^2 = (6)^2 + 2a \times 8$$

$$a = \frac{2^2 - 6^2}{2 \times 8} = -2 \text{ ms}^{-2}$$

Projectiles

We will now extend this theory to deal with the specific two-dimensional case where a particle has been launched in some way. In such a case the particle is called a *projectile.* A cannon ball in flight is an example of *projectile*. In this chapter we will ignore the effect of air-resistance.



The path that the projectile takes is a parabola. The motion of a projectile in two dimensions may be resolved into horizontal and vertical components. Since we are ignoring air-resistance, there is only one force acting on the particle, once it has been launched. This is its weight. Weight acts downwards, so the vertical motion of the projectile is as if it were accelerating under gravity. The particle has an initial vertical velocity given to it by the method of projection. This velocity is directed upwards. The effect of gravity is initially to cause the projectile to slow down. At some time the object will cease travelling upwards coming to a moment of instantaneous rest after which gravity will cause it to accelerate back towards the surface of the earth. Throughout its flight, horizontally, the projectile is moving with constant speed. This horizontal speed is also the result of an impulse given to it by the method of projection. Once the projectile has been launched this speed does not change because we are assuming that there is no air-resistance, so there is nothing to slow it down. (This is an application of Newton's first law, which states that every object remains at rest or travelling in a straight-line with constant velocity unless acted upon by a resultant force.) Thus to solve problems with projectiles we can resolve in the horizontal and vertical directions separately.

For the horizontal motion of a projectile, assumed to be at constant speed, we use the equation

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

For vertical motion of a projectile under gravity, we use the equations of uniform acceleration

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

where

v = final velocity

u = initial velocity

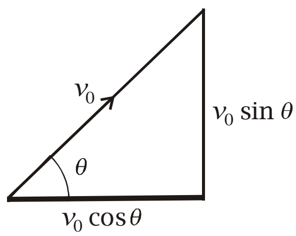
s = displacement

t = time taken

a = constant acceleration.

A particle travels upwards because it is given at the instant that it begins its flight an initial velocity v_0. For the cannon ball, this arises from the explosion in the barrel. For a golf ball the upward motion would arise from the impulse given to it by being struck by the golf-club. At no point after the projectile has been launched is it "pushed" or "pulled" by any force other than gravity. In these cases the projectile is not a rocket that carries with it its own means of propulsion. At all times it is accelerating towards the earth and it only continues to travel upwards until the gravitational force has brought its initial velocity to zero.

The initial velocity v_0 of the projectile is resolved into horizontal and vertical components. The particle is projected at an angle θ to the horizontal.



As the above diagram indicates the horizontal component of the velocity is

horizontal speed = $v_0 \cos\theta$

The vertical component of the velocity is

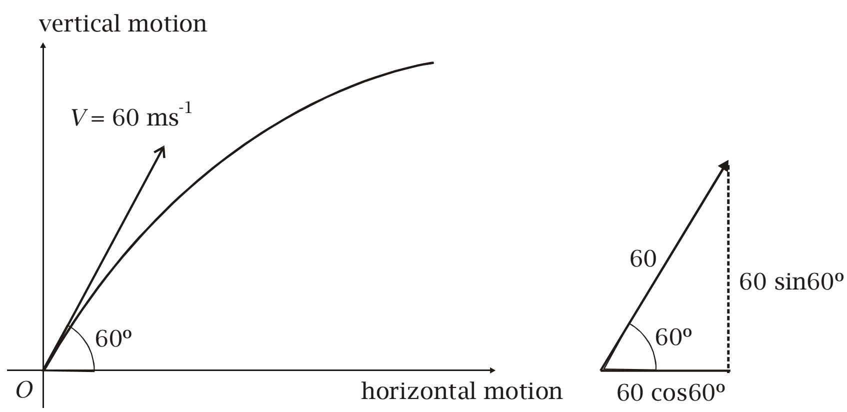
initial velocity upwards = $v_0 \sin\theta$

The projectile will reach its maximum height when the vertical velocity = 0. The total time of flight is twice the time taken to reach the maximum height.

Example (2)

A particle is projected with a speed of $V = 60 \text{ ms}^{-1}$ at an angle of 60° to the horizontal. Find its speed and the angle its velocity makes with the horizontal after 2 s.

Solution



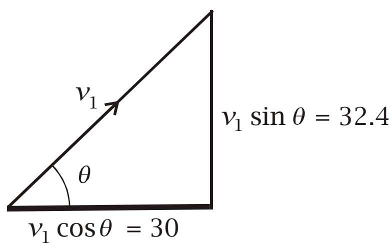
The vertical component of the velocity is given by

$v = u + at$

Here, initial vertical velocity, $u = 60\sin 60 = 60 \times \sqrt{3}/2 = 30\sqrt{3}$. Hence when $t = 2$

$v(2) = 30\sqrt{3} - 9.8 \times 2 = 32.4 \text{ ms}^{-1}$ (3 s.f.)

The horizontal component of the velocity is $60\cos 60 = 60 \times 1/2 = 30 \text{ ms}^{-1}$. This remains constant throughout the flight of the particle. So when $t = 2$ the particle is moving upwards with a speed of 32.4 ms^{-1} and horizontally with a speed of 30 ms^{-1}.



At this time its speed, which is the magnitude of the velocity, is given by

$|v_1| = \sqrt{32.4..^2 + 30^2} = 44.1 \text{ ms}^{-1}$ (3 s.f.)

The direction of the velocity is given by

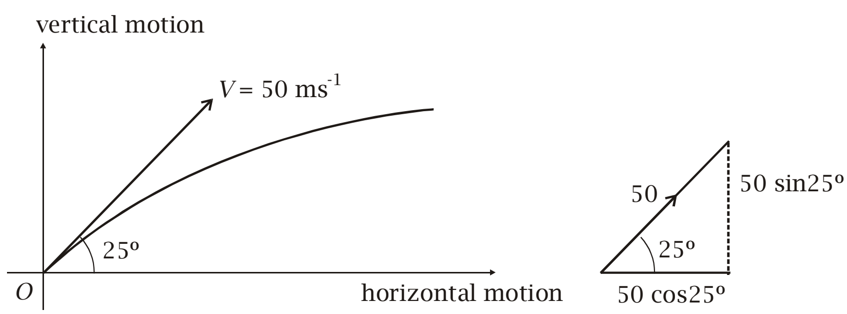
$\tan\theta = \dfrac{32.4..}{30}$

$\theta = \tan^{-1}\left(\dfrac{32.4..}{30}\right) = 47.2$ $(0.1°)$

Example (3)

A particle is projected with a velocity of 50 ms^{-1} at an angle of $25°$ to the horizontal. What is the greatest height of the projectile above the point of projection?

Solution



When the object reaches its greatest height the vertical component of its velocity is 0. To find the height we use the appropriate equation of uniform acceleration $v^2 = u^2 + 2as$.

Substituting $v = 0$, $u = 50\sin 25$, $a = -9.8$ we obtain

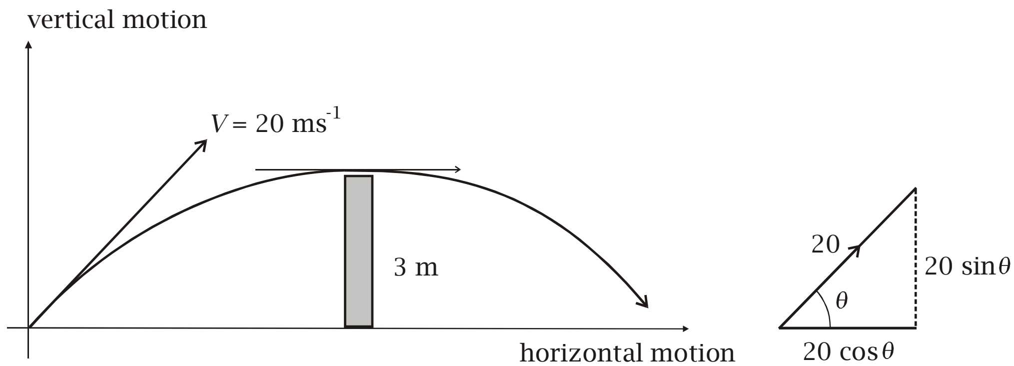
$$0 = (50\sin 25)^2 - 2\times 9.8\times s$$

$$s = \frac{(21.1..)^2}{2\times 9.8} = 22.8 \text{ m } (3 \text{ s.f.})$$

Example (4)

A golf ball is struck so that it just passes over a wall 3 m high, at which time it is moving horizontally. The initial speed of projection is 20 ms^{-1}. What is the angle of projection?

Solution



Let the initial angle of projection be θ.

To solve this problem we must first find the time at which the particle passes over the wall. This is found as follows.

$$v = u + at$$

$$0 = 20\sin\theta - 9.8\times t$$

$$t = \frac{20\sin\theta}{9.8}$$

We now substitute into one of the other equations of uniform acceleration to obtain an equation in θ. Into $s = ut + \frac{1}{2}at^2$ we substitute $s = 3$, $t = \frac{20\sin\theta}{9.8}$ and the initial vertical component of the velocity, which is $u = 20\sin\theta$, to obtain

$s = ut + \frac{1}{2}at^2$

$3 = 20\sin\theta\left(\frac{20\sin\theta}{9.8}\right) + \frac{1}{2}\times(-9.8)\times\left(\frac{20\sin\theta}{9.8}\right)^2$

$3 = \frac{400\sin^2\theta}{9.8} - \frac{400\sin^2\theta}{2\times 9.8}$

$3 = \frac{400\sin^2\theta}{2\times 9.8}$

$\sin^2\theta = \frac{3\times 2\times 9.8}{400}$

$\theta = 22.6^\circ$ (nearest 0.1°)