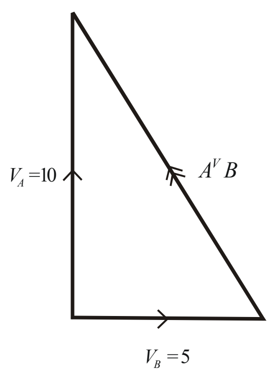
Relative Motion

Imagine two ships travelling. Ship *A* is travelling at 10 kmh^{-1} due North and ship *B* is travelling at 5 kmh^{-1} due East. Suppose you are standing on the deck of ship *B* and look at ship *A*. You will not see ship *A* travelling a 10 kmh^{-1} due North, because you are yourself travelling due North. What you see is the velocity of *A* relative to *B*. We use the symbol $A^V B$ for this.



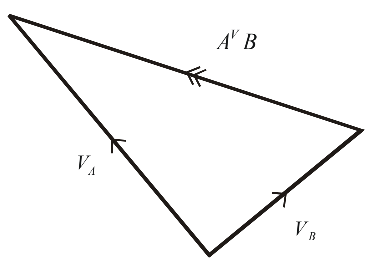
The example illustrates that we would expect the velocity of *A* relative to *B* ($A^V B$) to be the vector joining the tip of *B* to *A*.

Using the addition law for vectors

$V_B + A^V B = V_A$

$\therefore A^V B = V_A - V_B$

In general objects will be traveling in a certain direction (given by a bearing) and at a certain speed.



Thus, the relative velocity of A to B is the vector joining the tip of V_B to the tip of V_A

Further: $A^V B = V_A - V_B$

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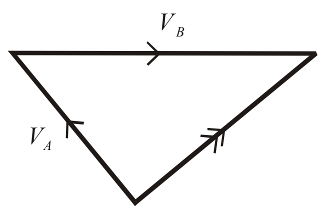
The meaning of $A^{V}B$ is that A appears to B to be moving with a velocity in this direction and of this magnitude – that is, this is what B sees by assuming that he is stationary.

When solving problems involving relative velocities:

1. Convert all velocities to component form – i.e. using **i** , **j** notation or equivalent.

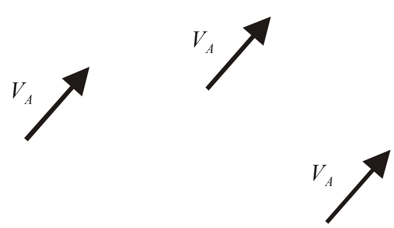
2. Use physical intuition to draw a correct diagram. Remember $A^{V}B$ is the vector joining B to A, so a diagram like this:



Does not give a relative velocity. This diagram shows the resultant of adding V_A to V_B.

3. Problems involving velocities are not direct problems involving distances. A diagram showing relative velocities does not show positions and distances and if these are required a separate diagram should be drawn.

Velocities are general vectors not position vectors. This means that in a diagram they do not have to be anchored anywhere. They can be shifted about. If they retain the same direction and magnitude then they are the same vector.



These are all the same vector – there is no origin in this diagram.

Example 1.

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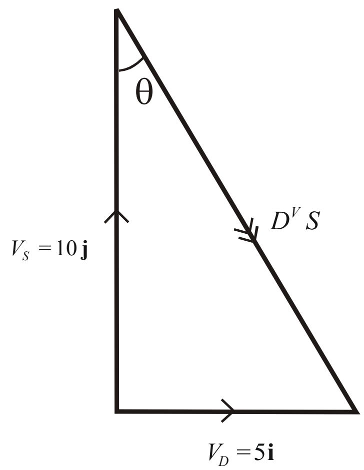
A ship is travelling at 10 kmh^{-1} due North and a dove is travelling at 5 kmh^{-1} due East. What is the apparent speed & direction of the dove to an observer on the ship?

Solution



$V_S = 10\underline{\mathbf{j}}$

$V_D = 5\ \underline{\mathbf{i}}$

$D^V S = V_D - V_S = 10\underline{\mathbf{j}} - 5\underline{\mathbf{i}}$

Apparent speed = $\left| D^V S \right| = \sqrt{10^2 + 5^2} = \sqrt{125}$

$$\theta = \tan^{-1}\left(\frac{5}{10}\right) = \tan^{-1}\left(0.5\right) = 26.6°$$

Bearing $z = 90 + \theta = 90 + 26.6 = 116.6°$

Example 2

A female skater is travelling at 3 ms^{-1} on a bearing of 70° and her partner is travelling at 4 ms^{-1} on a bearing of 140°. What is the velocity of the female skater relative to the male skater?
Solution

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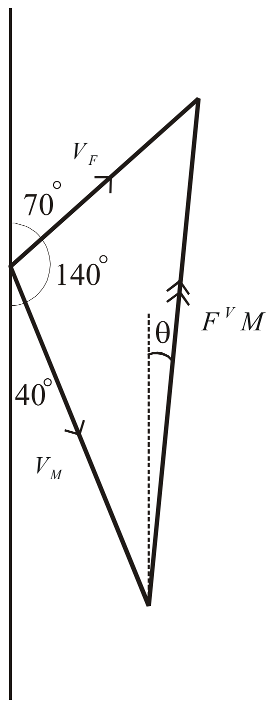

$V_F = 3\sin 70\,\underline{\mathbf{i}} + 3\cos 70\,\underline{\mathbf{j}}$

$V_M = 4\sin 40\,\underline{\mathbf{i}} - 4\cos 40\,\underline{\mathbf{j}}$

$F^V M = V_F - V_M$

$= (3 \sin 70\ \underline{\mathbf{i}} + 3 \cos 70\ \underline{\mathbf{j}}) - (v \sin 40\ \underline{\mathbf{i}} - 4 \cos 40\ \underline{\mathbf{j}})$

$= (3 \sin 70 - 4 \sin 40)\ \underline{\mathbf{i}} + (3 \cos 70 + 4 \cos 40)\ \underline{\mathbf{j}}$

$= 0.2479\ \underline{\mathbf{i}} - 4.0902\ \underline{\mathbf{j}}$

Apparent speed $= \sqrt{0.2479^2 + 4.0902^2} = 4.1 ms^{-1}$

Bearing $= \theta = \tan^{-1}\left(\frac{0.2479}{4.0902}\right) = 3.3°\left(\textit{Nearest } 0.1°\right)$

In the following example, you are given the relative velocity, and one other velocity and asked to find the other velocity. Use the equation $A^V B = V_A - V_B$, substitute in, and solve.

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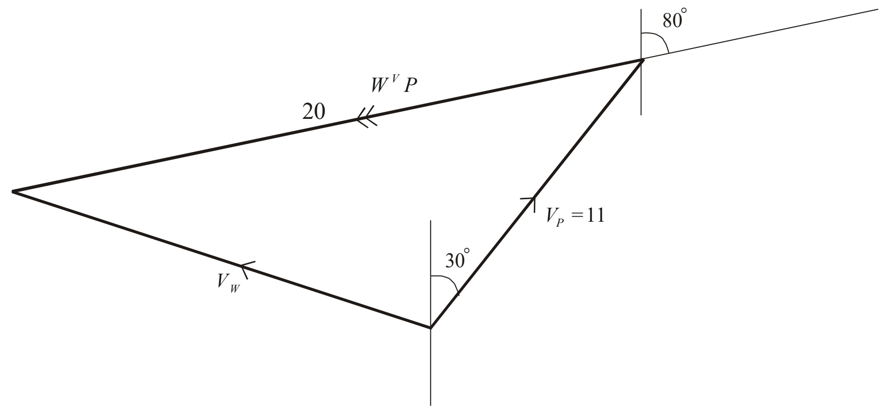
Example 3.

A toy plane flies at 11 kmh^{-1} on a bearing of 30°. The wind appears to be coming from 80° at 20 kmh^{-1}. What is the real velocity of the wind?



The apparent velocity of the wind is the velocity of the wind relative to the toy plane.

$W^V P = V_W - V_P$

$\therefore -20 \sin 80\, \underline{\mathbf{i}} - 20 \cos 80\, \underline{\mathbf{j}} = V_W - (11 \sin 30\, \underline{\mathbf{i}} + 11 \cos 30\, \underline{\mathbf{j}})$

$V_W = (-20 \sin 80 + 11 \sin 30)\, \underline{\mathbf{i}} - (20 \cos 80 - 11 \cos 30)\, \underline{\mathbf{j}}$

$= -14.196\, \underline{\mathbf{i}} + 6.053\, \underline{\mathbf{j}}$

speed $= |V_W| = \sqrt{14.196^2 + 6.053^2}$

$= 15.4$ kmh^{-1}

$\theta = \tan^{-1}\left(\frac{14.196}{6.053}\right) = 66.9°$

bearing = 360 – 66.9 = 293.1°

Interception

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If one object is moving faster than another it may not be possible to set a collision course. However, in this section we deal only with problems where a collision is possible.

If one object is to intercept another, then the relative velocity of one to the other lies along the straight line joining them.

You may be asked to find the collision course.

To solve the problem, begin by drawing a diagram showing the relative positions of the two objects. Then the line joining them is the line along which the relative velocity of one object to the other will lie. Draw a separate diagram showing the velocities of the two objects and their relative velocity acting along this line. The specific problem will then be soluble by substituting into the equation:

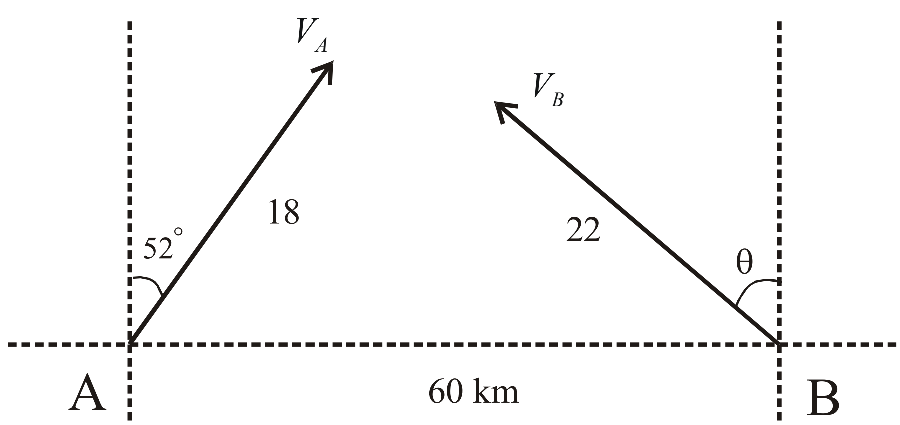
$A^{V}B = V_A - V_B$

Example 1.

A ship *A* lies 60 km to the west of a second ship *B*. *A* is travelling at 18 kmh^{-1} on a bearing of 52°. The ship *B* can travel at 22 kmh^{-1}. Find the course that *B* should set in order to intercept A. Find also the time taken to interception.

Solution

Diagram of positions



The relative velocity of B to A lies along the line joining B to A; hence, the diagram of velocities is

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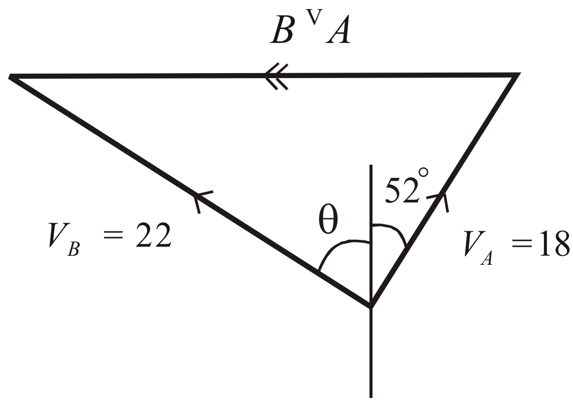

Thus:

$B\,^{V}A = V_B - V_A$

Let $B^{V}A = x\,\underline{\mathbf{i}}$

Then: $-x\,\underline{\mathbf{i}} = -22 \sin\theta\,\underline{\mathbf{i}} + 22\cos\theta\,\underline{\mathbf{j}} - (18 \sin 52\,\underline{\mathbf{i}} + 18 \cos 52\,\underline{\mathbf{j}})$

Resolving vertically:

$0 = 22 \cos\theta - 18 \cos 52$

$$\cos\theta = \frac{18\cos 52}{22}$$

$\theta = 59.8°$ (0.1°)

Bearing = 360 – 59.8 = 300.2°

Resolving horizontally:

$-x = -20 \sin\theta - 15 \sin 48$

$x = 20 \sin 59.8 + 15 \sin 48 = 32.0$ kmh^{-1}

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{60}{32.0} = 1.87 \text{ hours}$$

Example 2.

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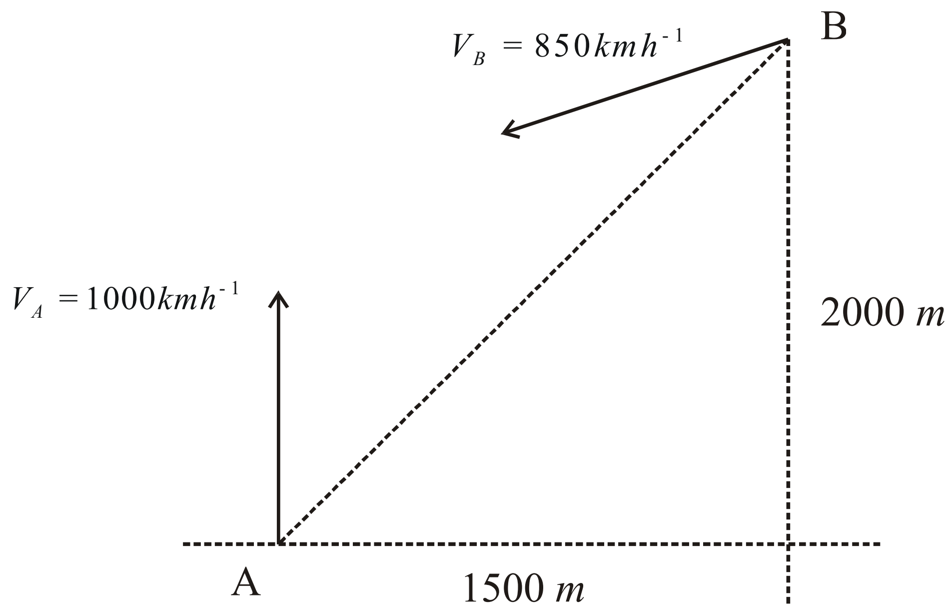
This illustrates the case where the resultant velocity does not lie along a "nice" horizontal or vertical path.

A fighter plane, A, is traveling north at 1000 kmh^{-1}. Another fighter, B, which can travel at 850 kmh^{-1} wishes to intercept it. It is currently positioned 1500km to the East and 2000km to the North of A. Find the course B should plot & the time taken to interception.

Solution

The following diagram shows the positions of the fighters.



The relative velocities will lie along the line joining *B* to *A*; hence, we calculate the bearing by

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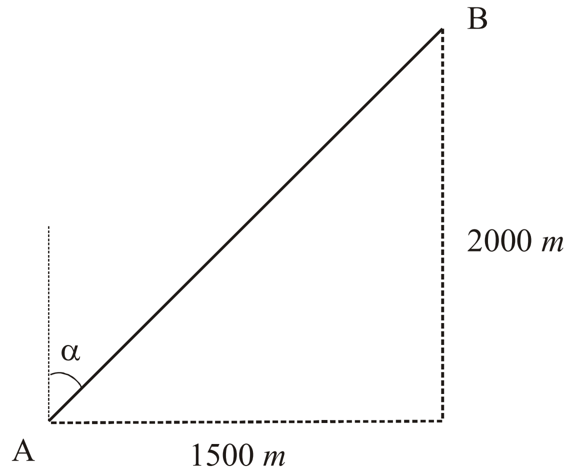
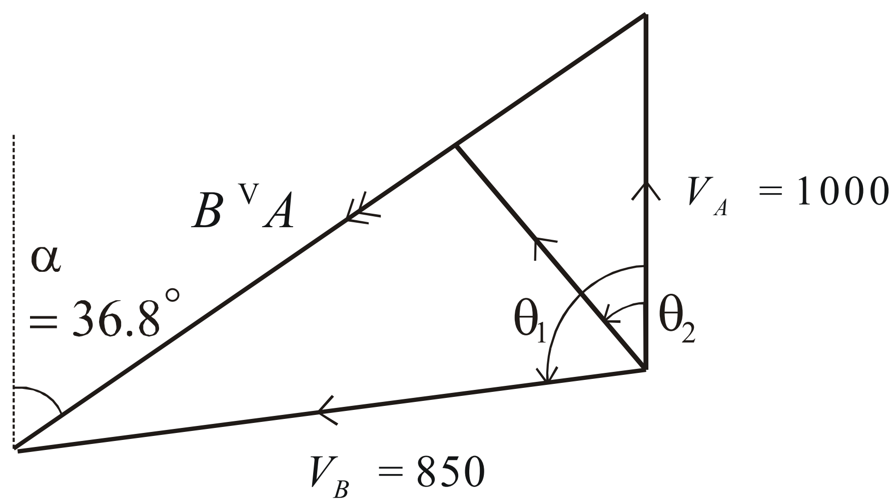


$\alpha = \tan^{-1}\left(\dfrac{1500}{2000}\right) = 36.86°$

The diagram of the velocities is



The diagram shows that fighter B could intercept along two paths, with bearings θ_1 and θ_2. The shortest time will be given by the path with bearing θ_1.

$B^{V}A = V_A - V_B$

Let $|B^{V}A| = x$

Then

$$-x\sin 36.8\,\underline{\mathbf{i}} - x\cos 36.8\,\underline{\mathbf{j}} = -850\sin\theta\,\underline{\mathbf{i}} + 850\cos\theta\,\underline{\mathbf{j}} - 1000\,\underline{\mathbf{j}}$$

Note here we have put +850 cos θ **j** because if θ > 90 this will be a negative value, but we put –850 sin θ **i** because we show that the horizontal component of V_B is in the same direction as the horizontal component of $B^V A$.

Uncoupling: (Resolving horizontally and vertically separately):

1. $-x\sin 36.8 = -850\sin\theta$

2. $-x\cos 36.8 = 850\cos\theta - 1000$

From (1)

$$x = \frac{850\sin\theta}{\sin 36.8}$$

Substituting in (2)

$$\frac{-850\sin\theta}{\sin 36.8}\cos 36.8 = 850\cos\theta - 1000$$

$$-850\sin\theta\cot 36.8 = 850\cos\theta - 1000$$

$$-850\sin\theta \times 1.33^{\circ} = 850\cos\theta - 1000$$

$$-1133^{\circ}\sin\theta = 850\cos\theta - 1000$$

$$(1133)^2\sin^2\theta = (850)^2\cos^2\theta - 1700000\cos\theta + (1000)^2$$

$$(1133)^2\left(1-\cos^2\theta\right) = 850^2\cos^2\theta - 1700000\cos\theta + 1000^2$$

$$1133^2 - 1133^2\cos^2\theta = 850^2\cos^2\theta - 1700000\cos\theta + 1000^2$$

$$\left(850^2 + 1133^2\right)\cos^2\theta - 1700000\cos\theta + 1000^2 - 1133^2 = 0$$

$$2006944\cos^2\theta - 1700000\cos\theta - 284444 = 0$$

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$$\cos\theta = \frac{1700000 \pm \sqrt{1700000^2 + 4 \times 2006944 \times 284444}}{2 \times 2006944}$$

$$\cos\theta = \frac{1700000 \pm 2274522}{2 \times 2006944}$$

$\cos\theta = 0.990$ or -0.143

$\theta_2 = 8.03^\circ$ $\qquad$ $\theta_1 = 98.23^\circ$

The shortest approach is given by $\theta_1 = 98.23°$

Horizontal component $= X \sin 36.8$

$$= \frac{850 \sin\theta}{\sin 36.8} \sin 36.8$$

$$= 850 \sin\theta$$

= 850 sin 98.23

= 841.2 kmh^{-1}

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{1500}{841.2} = 1.78 \text{ hrs}$$

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