

# Representation of Forces by Vectors

A vector is a physical quantity with magnitude (size) and direction.

A scalar is a physical quantity with only magnitude.

Distance travelled is a scalar. Displacement, which is the term for distance travelled in a certain direction, is a vector.

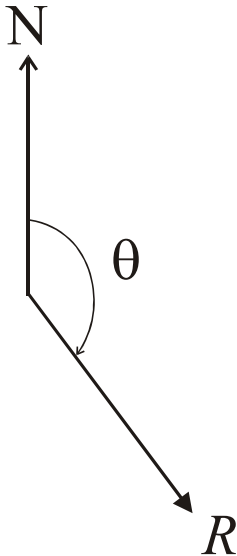
For example, 5 metres travelled North is the same distance as 5 metres travelled South, but it is in an opposite direction, so it is a different displacement.

Speed is a scalar; velocity is a vector. Velocity is speed in a given direction. Thus, when an object moves in a circle with a constant speed, its velocity is always changing because it is always changing direction.

Other vector quantities include force, acceleration and momentum.

Vectors can be expressed in terms of magnitude and direction.

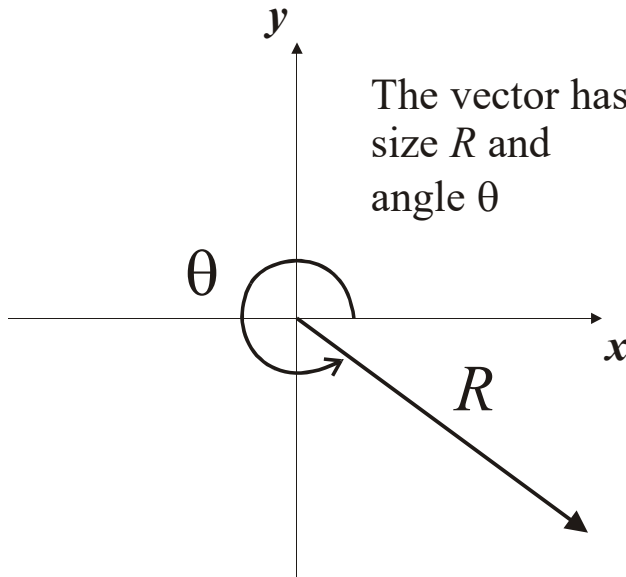
In navigation a vector is often specified by means of a size, or range, and a bearing, which is an angle less than  $360^\circ$  measured clockwise from the north direction.



The vector is defined by its range,  $R$ , and its bearing,  $\theta^\circ$ .



In coordinate geometry a vector may be specified by a size and an angle, measured anticlockwise from the positive  $x$ -axis. We call this representation its polar coordinates.



Vectors can also be resolved into components.

In two dimensions it is often useful to resolve vector quantities into horizontal and vertical components. When we want to be clear that we are discussing only the size of a vector,  $\mathbf{R}$ , we use the modulus symbol:

$$|\mathbf{R}| = \text{size of the vector } \mathbf{R}$$

But often the modulus symbol is dropped when the meaning is clear.

When we want to be clear that the quantity under discussion is a vector, we underline the symbol:

$$\underline{R} = \text{the vector } R$$

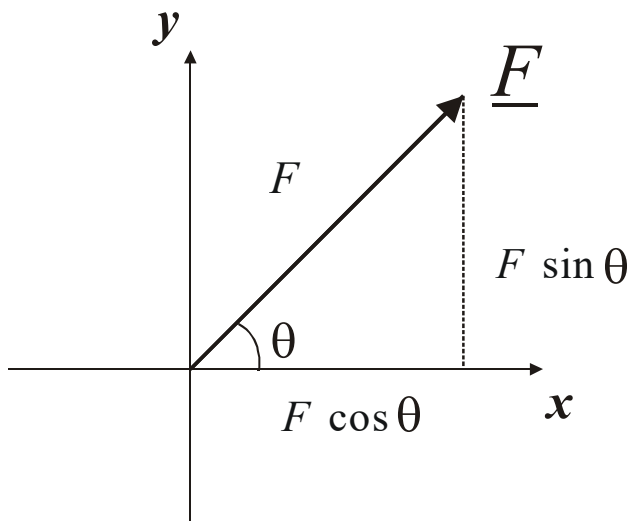
In textbooks vectors are printed with bold type.

$$\mathbf{R} = \text{the vector } R$$

Here we follow the convention that vectors will be both in bold and underlined.

To resolve vectors into their horizontal and vertical components we use trigonometric relationships.





If a force  $\underline{F}$  has magnitude  $|F|$  and acts in a direction  $\theta$ , then its horizontal and vertical components are  $|F|\cos\theta$  and  $|F|\sin\theta$  respectively.

As already remarked, we usually drop the modulus sign and write  $F\cos\theta$  and  $F\sin\theta$ .

Sometimes we wish to represent the horizontal and vertical components of a vector in a single line. In order to do this we need some way of keeping the horizontal and vertical components separate. To achieve this there are several methods:

### Column Form

We can write the components in a vertical column

$$\begin{pmatrix} F\cos\theta \\ F\sin\theta \end{pmatrix}$$

The convention is that the horizontal component comes first.

### Row Form

We can write the components as an ordered pair (as a row vector)

$$(F\cos\theta, F\sin\theta)$$



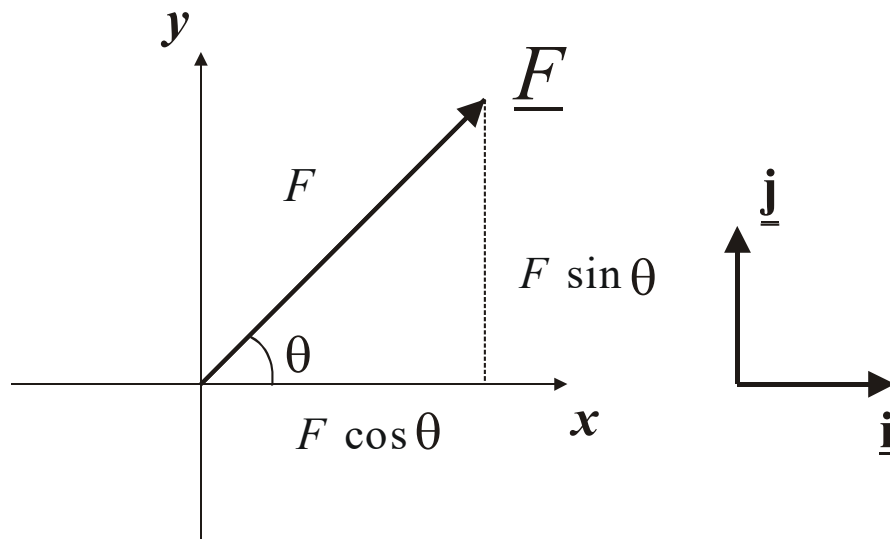
The horizontal component comes first.

### **i, j notation**

We use the symbols **i** and **j** to keep the two components apart. These symbols do not stand for any physical quantities, but are “dummy” symbols for keeping the horizontal and vertical components of a vector apart. They represent directions.

**i** is a unit vector in the horizontal direction and **j** is a unit vector in the vertical direction.

In summary, a vector **F** can be represented in its component form in three ways:



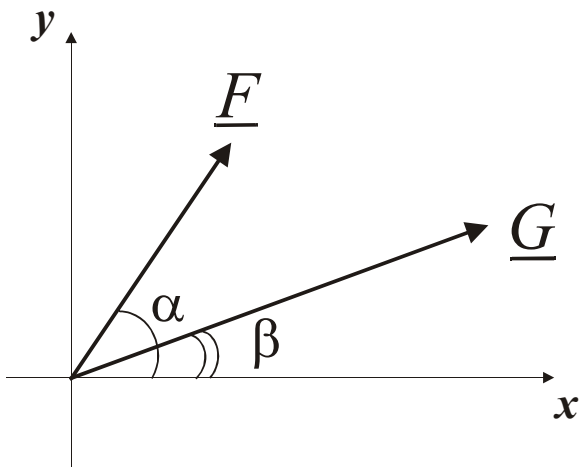
$$\begin{aligned}\underline{\mathbf{F}} &= \begin{pmatrix} F \cos \theta \\ F \sin \theta \end{pmatrix} \\ &= (F \cos \theta, F \sin \theta) \\ &= F \cos \theta \underline{\mathbf{i}} + F \sin \theta \underline{\mathbf{j}}\end{aligned}$$

### **Addition of Vectors**

Forces are vectors and obey the rules for the addition of vectors.

When we add vectors we simply add the horizontal and vertical components.



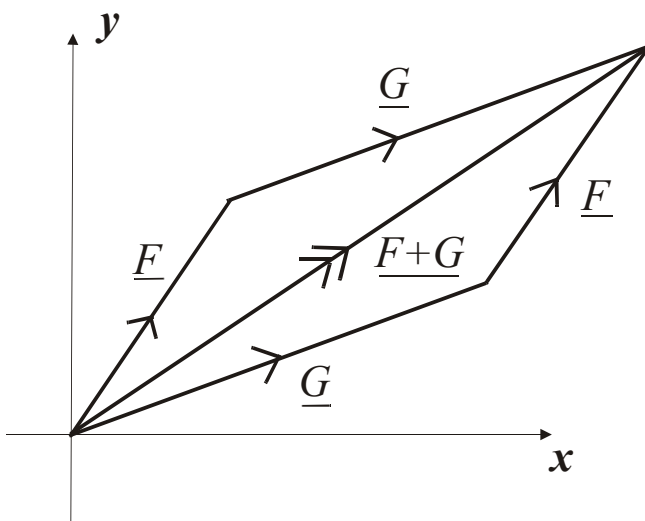


If  $\underline{\mathbf{F}} = \begin{pmatrix} F \cos \alpha \\ F \sin \alpha \end{pmatrix}$  and  $\underline{\mathbf{G}} = \begin{pmatrix} G \cos \beta \\ G \sin \beta \end{pmatrix}$

Then  $\underline{\mathbf{F}} + \underline{\mathbf{G}} = \begin{pmatrix} F \cos \alpha + G \cos \beta \\ F \sin \alpha + G \sin \beta \end{pmatrix}$

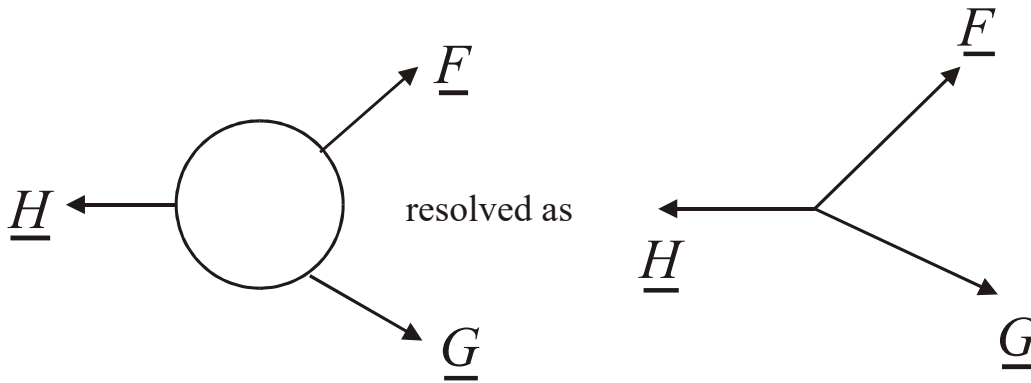
The vector  $\underline{\mathbf{F}} + \underline{\mathbf{G}}$  is called the resultant of  $\underline{\mathbf{F}}$  and  $\underline{\mathbf{G}}$ .

The rule for adding vectors means that the resultant obeys the parallelogram rule: the resultant is the diagonal of the parallelogram formed with the vectors  $\underline{\mathbf{F}}$  and  $\underline{\mathbf{G}}$  as sides.



Since a vector is specified by a size and a direction any two parallel sides of the parallelogram represent the same vector.

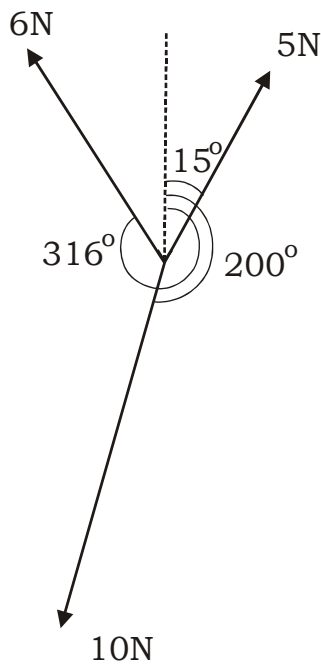
Although forces can cause objects to rotate their effect on the linear speed and direction of the object does not depend on where on the object they act. So any collection of forces acting on an object can be added (or “resolved”) as if they all applied to a single point.



When resolving three or more vectors, or forces, add all their components.

#### Example

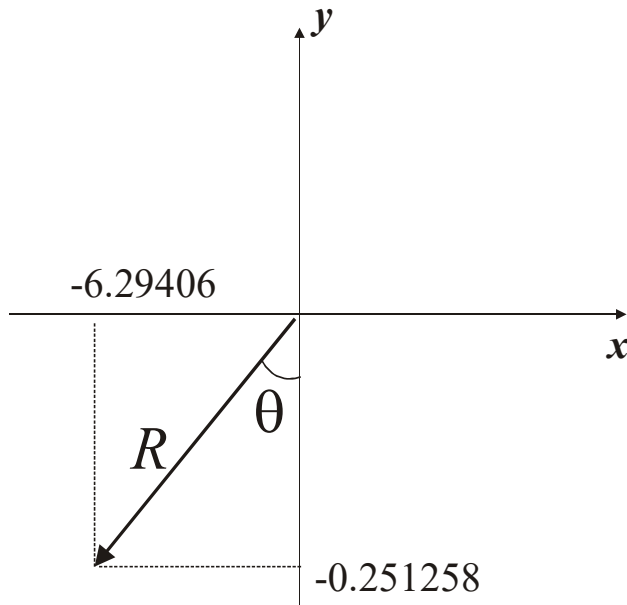
Find the resultant of the three forces: 5N at a bearing of  $15^\circ$ ; 10N at a bearing of  $200^\circ$ ; 6N at a bearing of  $316^\circ$ .



Solution

Add the forces as column vectors

$$\text{Resultant} = \begin{pmatrix} 5 \cos 15 \\ 5 \sin 15 \end{pmatrix} + \begin{pmatrix} -6 \cos 44 \\ 6 \sin 44 \end{pmatrix} + \begin{pmatrix} -10 \cos 20 \\ -10 \sin 20 \end{pmatrix} = \begin{pmatrix} -6.29406 \\ -0.251258 \end{pmatrix}$$



$$|R| = \sqrt{(-6.29406)^2 + (-0.251258)^2}$$
$$= 6.30 \text{ (2.S.F.)}$$

$$\theta = \tan^{-1} \left( \frac{6.29406}{0.251258} \right) = 87.7^\circ \text{ (to } 0.1^\circ)$$

$$\text{Bearing} = 180 + 87.7 = 267.7^\circ$$

