Scalar multiples of a Poisson variable

If the interval over which a Poisson distribution is taken is doubled, then the mean of the Poisson distribution is doubled; likewise, if the interval is halved, then the mean is also halved.

If $X \sim Po(\mu)$ in the interval I

Then $X \sim Po(n\mu)$ in the interval nI

Example

An electrical relay (that is, an electrical switch) makes a connection within $1ns$ ($1\text{x}10^{-9}s$) of being switched on 99 out of every 100 trials. (i) Find the probability that in 300 trials the switch will fail to connect within $1ns$ on more than 4 occasions. (ii) Find also the probability that in 20 trials the switch fails to connect within $1ns$ on 0 occasions.

Answer

(i) In 100 trials we expect 1 failed connection, hence

$Y \sim Po(1)$ in 100 trials

Therefore, in 300 trials we expect 3 failures on average,

and if Y denotes the number of failures in 300 trials, then

$X \sim Po(3)$

Then, we require

$$P(X > 4) = 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3) - P(X = 4)$$

Now

$$P(X = 0) = e^{-3} = 0.049787$$

$$P(X = 1) = 3 \times P(X = 0) = 0.149361$$

$$P(X = 2) = \frac{3}{2} \times P(X = 1) = 0.224042$$

$$P(X = 3) = \frac{3}{3} \times P(X = 2) = 0.224042$$

$$P(X = 4) = \frac{3}{4} \times P(X = 3) = 0.168031$$

$$P(X \leq 4) = 0.815263$$

$$P(X > 4) = 0.184737 = 0.18 \ (2.\text{S.F.})$$

(ii) $Y \sim Po(1)$ in 100 trials

$X \sim Po(0.2)$ in 20 trials

$P(X = 0) = e^{-0.2} = 0.8187 = 0.82$ (2.S.F.)