Simultaneous equations in three unknowns

You should be familiar with the process of solving simulataneous equations in two unknowns by the methods of elimination and substitution, and also the graphical interpretation of this process.

In this unit we extend the idea of solving simulataneous equations to the situation where there are *three* unknown quantities.

A collection of three equations in three unknowns is called a *system* of simulataneous equations. For example

$$2x - 3y + z = 13 \qquad (1)$$
$$x + y - 2z = -1 \qquad (2)$$
$$3x - 2y + 2z = 18 \qquad (3)$$

is a system of simulateneous equations.

Not every system of simulataneous equations can be solved. This topic, of when a system of simultaneous equation can and cannot be solved, is dealt with in a separate unit. Here we are interested only in developing algebraic technique and extending the methods of elimination and substitution to slightly harder problems.

Thus, all the questions and examples in this unit may be assumed to have solutions.

The key to solving these equations is to reduce the three equations to two equations, and then solve those two equations by the usual method. This is typical of mathematics in general, which builds more and more complex methods and intuitions of simpler methods and concepts.

To continue with our example

$$2x - 3y + z = 13 \qquad (1)$$
$$x + y - 2z = -1 \qquad (2)$$
$$3x - 2y + 2z = 18 \qquad (3)$$

If we multiply the second equation by 2 and subtract from the first equation

$$2x - 3y + z = 13 \qquad (1)$$
$$2x + 2y - 4z = -2 \qquad (2) \times 2 = (4)$$
$$-5y + 5z = 15 \qquad (1) - (4) = (5)$$

we eliminate x from these two equations. Likewise, multplying the second equation by 3 and subtracting from the third equation eliminates x from these two equations.

$$3x - 2y + 2z = 18 \qquad (3)$$
$$3x + 3y - 6z = -3 \qquad (2) \times 3 = (6)$$
$$-5y + 8z = 21 \qquad (3) - (6) = (7)$$

Now we have a system of *two* simulataneous equations in *two* unknowns, so we have simplified the problem.

$$-5y + 5z = 15 \qquad (5)$$
$$-5y + 8z = 21 \qquad (7)$$

Subtacting these two equations gives

$$-3z = -6 \qquad (5) - (7)$$
$$z = 2$$

Now we can substitute back into (5)

$$-5y + 10 = 15$$
$$y = -1$$

Finally, by substituting into (2)

$$x - 1 - 4 = -1$$
$$x = 4$$