Single-sample sign test of a population median

A single-sample sign test is an example of a non-parametric test. It is used to test a hypothesis about a population median and its source is a set of ordinal level data. The use of this test is best demonstrated through example.

Example

Before the resignation of the headmaster, a random sample of 15 parents were asked to rate the performance of the school on a scale of 1 to 10. Six months after the headmaster's resignation the same questionnaire was put to another 12 parents selected at random. Before the resignation the median rating was 5, where 1 was the lowest possible score and 10 the highest. The values for the second survey were

6, 5, 8, 7, 4, 3, 6, 6, 9, 7, 8

Use a binomial sign test at the 5% significance level to determine whether the median rating has increased.

The hypotheses are

H_0: median = 5
H_1: median > 5

The characteristic of this example is that we have been given the value of the median from the previous sample but no further information. Consequently, the order of the values in the second sample cannot be compared with the order of the values in the first sample. Although the Wilcoxon signed ranks test enables us to use the information contained in the ranks of the data, here we begin by reducing the data to nominal level data, and we classify the data according to three classes, then perform a "simpler" test:

+ if the value is greater than 5.

– if the value is less than 5.

0 if the value is equal to 5.

Thus we score the data in our example as:

6	5	8	7	4	3	6	6	9	7	8
+	0	+	+	-	-	+	+	+	+	+

The data which give 0 scores are removed. Thus we have 9 + signs and 2 – signs. If the median is 5 then for this sample there should be an equal probability of obtaining a score greater than five (a+) to obtaining a score less than five (a-). Hence, the number of positive (or negative) signs follows a binomial distribution with a probability of a "success", $p = 0.5$.

If X = the no. of –ve signs

Then $X \sim B(11, 0.5)$.

The question, 'how unusual are just two –ve signs?' becomes, 'what is the probability of obtaining 2 or less than 2 –ve signs?'

$$\begin{aligned}
P(X \le 2) &= P(X=0) + P(X=1) + P(X=2) \\
&= {}^{11}C_0(0.5)^{11} + {}^{11}C_1(0.5)^{11} + {}^{11}C_2(0.5)^{11} \\
&= ({}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2)(0.5)^{11} \\
&= (1+11+55)(0.5)^{11} \\
&= 0.0327 \quad (3\ s.f.)
\end{aligned}$$

The critical value for this test is the significance level α=0.05

Hence test value < critical value

Therefore, we reject H_0 and accept H_1.

The parents really do think the school has improved.