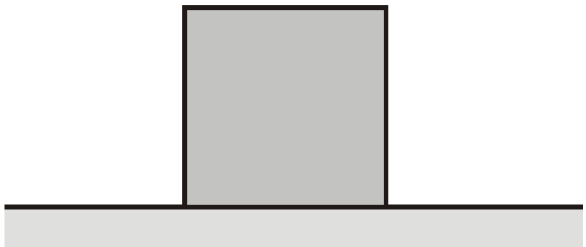
Sliding, toppling and suspending

Sliding and toppling

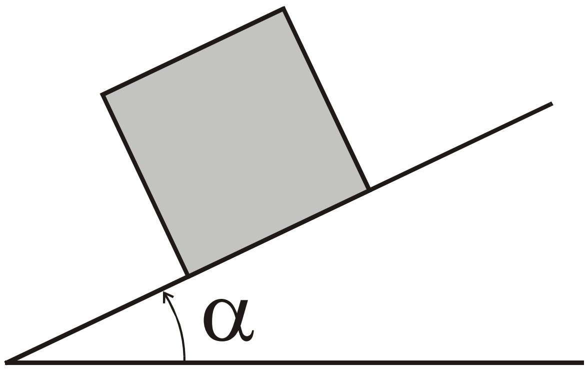
When an object lies on an inclined plane there are three things that could happen.

1. It remains art rest in equilibrium.
2. It slides down the surface.
3. It topples over.

Imagine an object placed on a level surface that is capable of being raised at one end, whilst the other remains fixed.

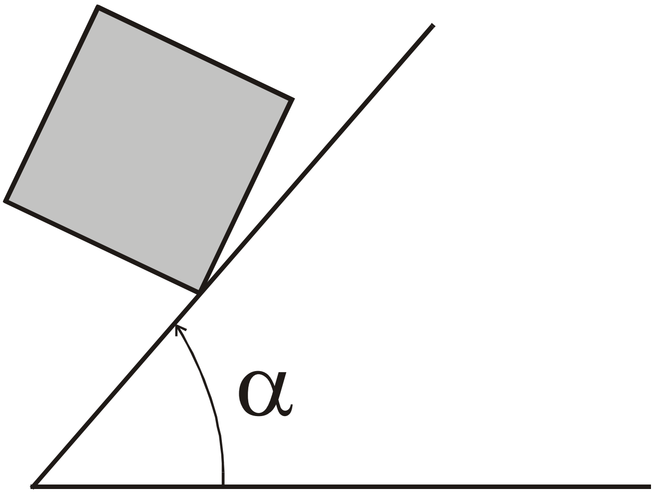


The surface is slowly tilted through an angle α .



Eventually it will not be able to remain at rest on the surface. The question is, as the angle α is increased, whether it will slide first or topple first.

In the following diagram the object is toppling.

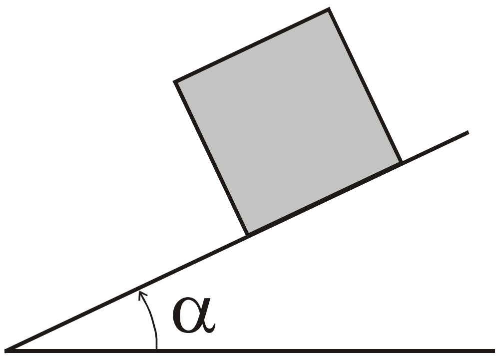


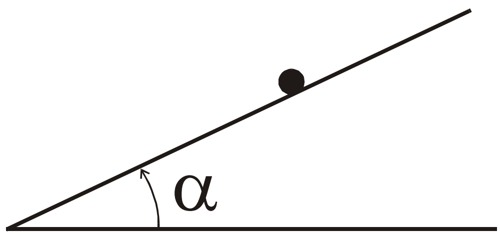
It will slide first if the friction between the object and the surface is not great enough to keep it in contact with the surface. If the surface is "smooth" it is more likely to slide first. Otherwise, when its centre of gravity no longer acts through the surface, then it will topple.

To determine whether the object slides or topples we need to find the critical values of α at which the object slides and topples respectively.

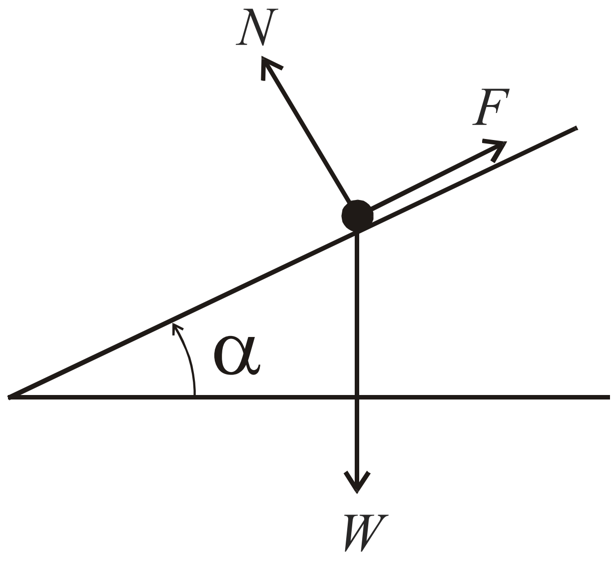
Sliding

In order to discuss the sliding case, it will be convenient to replace our object by a particle resting on the inclined surface.



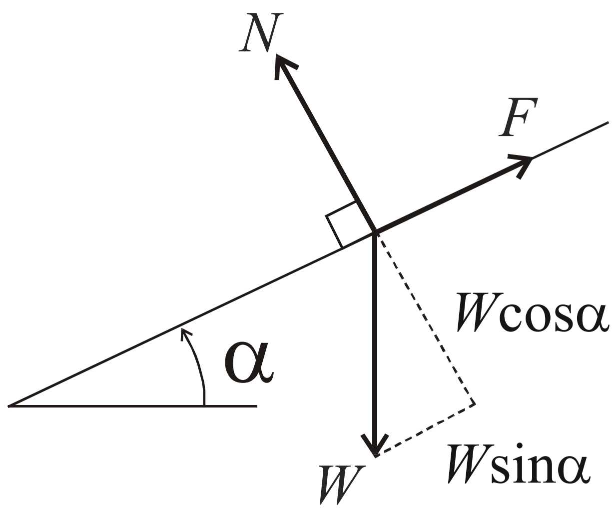


The forces acting on the particle are its weight, W, the normal reaction at the surface, N, and the friction, F, acting up the slope and preventing it from sliding.



Let us also imagine that the particle is just about to slide. This means that the component of the weight of the particle acting down the slope is just equal to the friction acting up the slope.

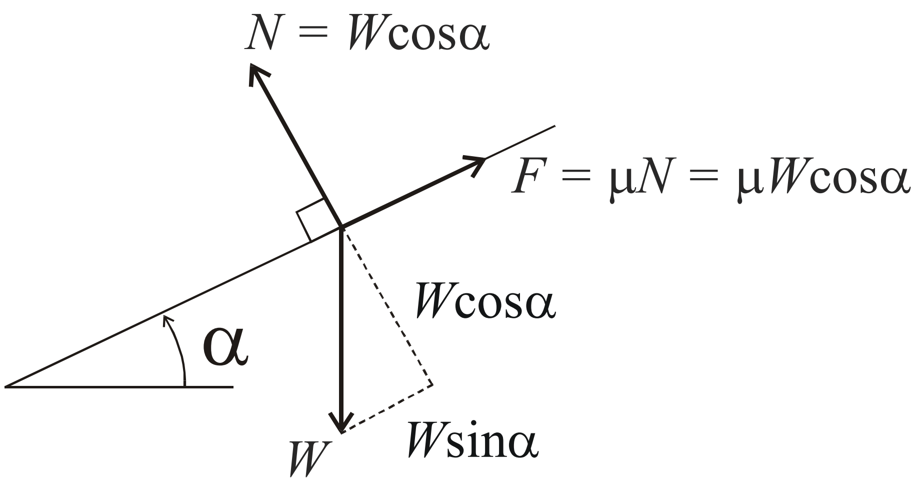
The component of the weight acting down the slope is $W \sin \alpha$, and the component of the weight acting through the surface, at right-angles to it is $W \cos \alpha$, by the usual laws of trigonometry.



The friction is related to the normal force in the usual way by

$F = \mu N$

where μ is the coefficient of friction, a number between 0 (perfectly smooth), and 1 (completely rough). Thus, to complete the diagram



It will just be about to slide when the down-slope component of the weight equals the friction. That is, when

$W \sin\alpha = \mu W \cos\alpha$

Rearranging gives

$$\frac{W \sin\alpha}{W \cos\alpha} = \mu$$

So the object will be just about to slide when

$\tan\alpha = \mu$

It will slide when the component of the weight acting down the slope is greater than the friction acting up the slope; that is when

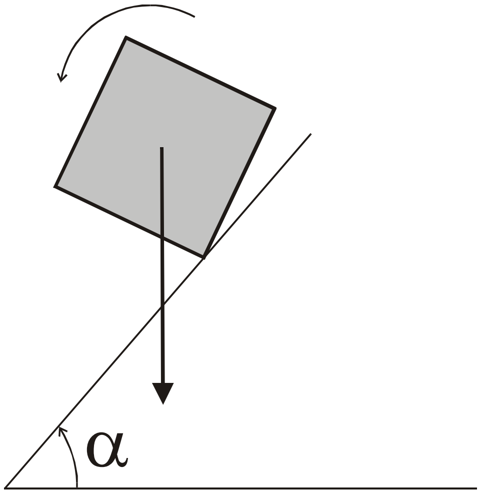
$W \sin\alpha > \mu W \cos\alpha$

Hence

$\tan\alpha > \mu$

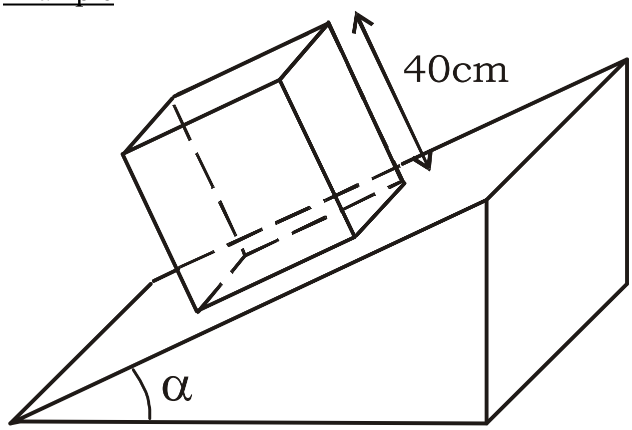
Toppling

An object will topple when its centre of gravity is such that its weight does not act through the surface of contact with the inclined plane.



This is a geometric problem and requires knowledge first of the position of the centre of gravity of the object in question, so we will tackle this topic by means of an illustration.

Example



A solid block of wood of height 40*cm* with square base $12cm^3$ lies on a rough plane. The plane is inclined at an angle α to the horizontal. The coefficient of friction between the plane and the wooden block is μ.

(a) Given that the block is in equilibrium show that $\tan\alpha \le \mu$ and $\tan\alpha \le \frac{3}{10}$

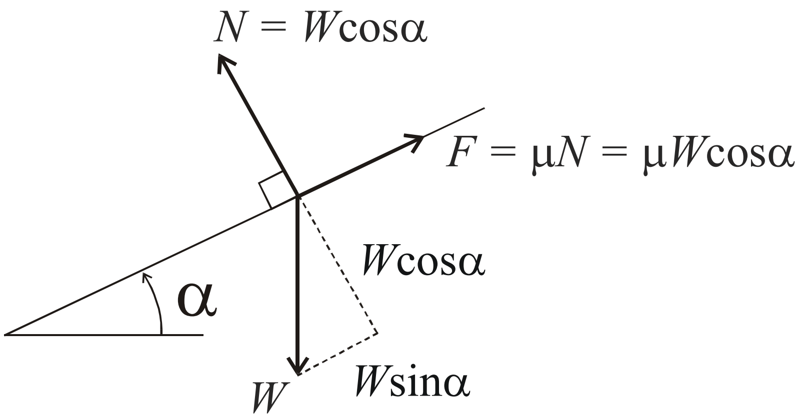
(b) State what happens if (i) $\mu = \frac{1}{4}$ and $\tan\alpha = \frac{1}{2}$; (ii) $\mu = \frac{2}{5}$ and $\tan\alpha = \frac{3}{10}$

Answer

(a) The block is neither sliding nor toppling. Since it is not sliding, the friction is sufficient to keep it in equilibrium. We showed above that for any object, for it to slide we must have hence

$\tan\alpha > \mu$

Redrawing the diagram to confirm this

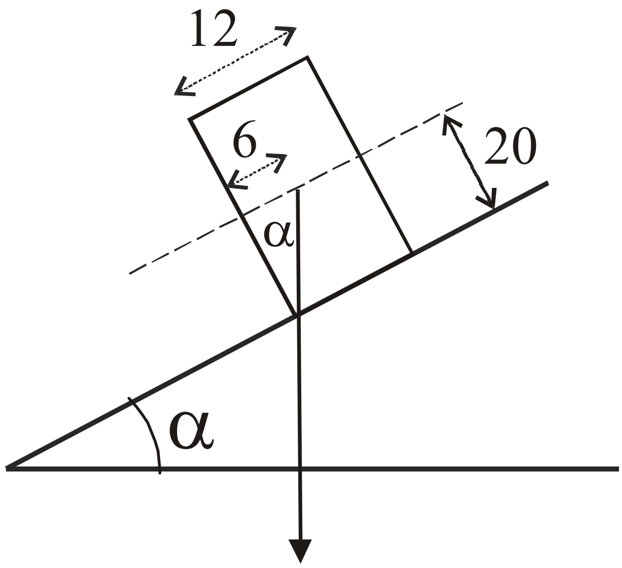


We have

$W\sin\alpha \le \mu W\cos\alpha$

$\therefore \tan\alpha \le \mu$

We now illustrate ideas about toppling. Suppose the object was just about to topple, then the following situation would obtain



The diagram shows how the centre of gravity would be just acting through the corner of the object. If the angle is increased any further, then it will topple. At this point the angle, α, is given by trigonometry as

$$\tan\alpha = \frac{6}{20} = \frac{3}{10}$$

Since the object is not toppling, the angle must be

$$\tan\alpha \le \frac{3}{10}$$

Hence, since the object is neither sliding nor toppling, $\tan\alpha \le \mu$ and $\tan\alpha \le \frac{3}{10}$ as required.

(b) (i) If $\mu = \frac{1}{4}$, then $\tan\alpha = \frac{1}{2} > \mu$, so it slides.
Since $\tan\alpha = \frac{1}{4} < \frac{3}{10}$, it does not topple.
That is, it slides but does not topple.

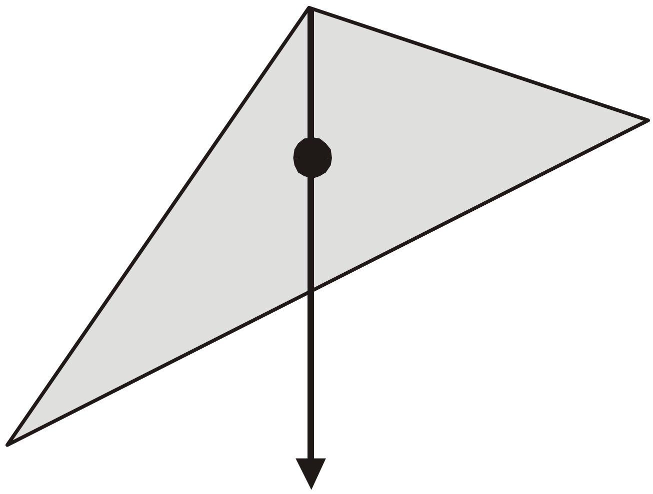
(ii) $\mu = \frac{2}{5}$, $\tan\alpha = \frac{3}{10}$ so it just about to topple.

Since $\tan\alpha < \mu$ it does not slide.
That is, it is about to topple, but will not slide.

Suspended objects

When an object is suspended from a point it hangs under the effect of gravity.

$\therefore$ The line joining that point to the centre of gravity must be vertical (or rather lie in the direction of the field)



This geometric fact can be used to solve problems involving the position of suspended bodies.

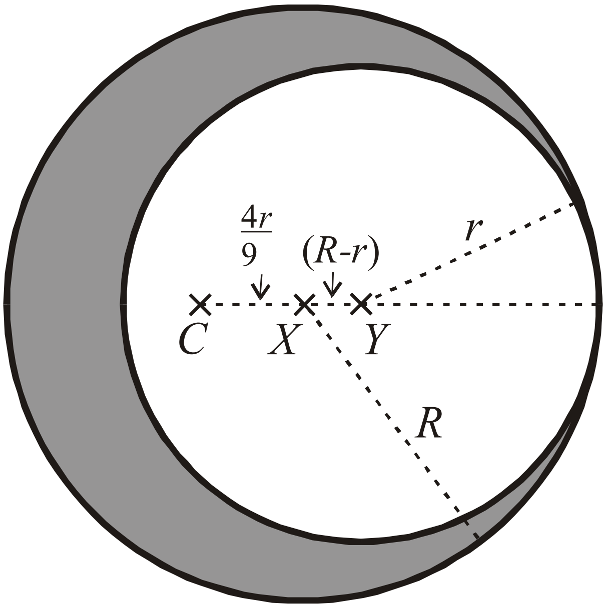
Example

A circular disc of uniform density has centre X and radius R. A circular hole is cut into it, with radius r and centre Y, where $r < R$. The lamina forms a crescent shape. In addition, we have $XY = R - r$, and the centre of gravity lies at a distance $\frac{4r}{9}$ from X.

(i) Show that $R = \frac{5r}{4}$;

(ii) Given that the lamina is suspended from a point on its outer rim that lies on the perpendicular to XY through X, find the angle which XY makes with the vertical.

Note - these topics of suspending, toppling and sliding depend on understanding of centres of gravity, hence the two parts to the question. However, simply knowing that the centre of gravity lies $4r/9$ from the centre of the disc enables you to solve the first part of the question.



(i) Taking moments about the centre of gravity of the disc (which will be our pivot point)

$$\begin{pmatrix} \text{mass of} & & \text{distance of} \\ \text{disc} & \times & \text{c.g. of disc} \\ & & \text{from pivot} \end{pmatrix} = \begin{pmatrix} \text{mass of} & & \text{distance of} \\ \text{crescent} & \times & \text{c.g. of crescent} \\ & & \text{from pivot} \end{pmatrix} - \begin{pmatrix} \text{mass of} & & \text{distance of} \\ \text{hole} & \times & \text{c.g. of hole} \\ & & \text{from pivot} \end{pmatrix}$$

Let $\rho =$ density of the uniform lamina, then this formula gives

$$\left(\pi R^2 p \times 0\right) = \left(\left(\pi R^2 - \pi r^2\right)\rho \times \frac{4r}{9}\right) - \left(\pi r^2 \rho \times |XY|\right)$$

But we are given that $|XY| = R - r$, hence

$$\left(\pi R^2 p \times 0\right) = \left(\left(\pi R^2 - \pi r^2\right)\rho \times \frac{4r}{9}\right) - \left(\pi r^2 \rho \times (R - r)\right) = 0$$

$$\left(R^2 - r^2\right)\frac{4}{9} - r\left(R - r\right) = 0$$

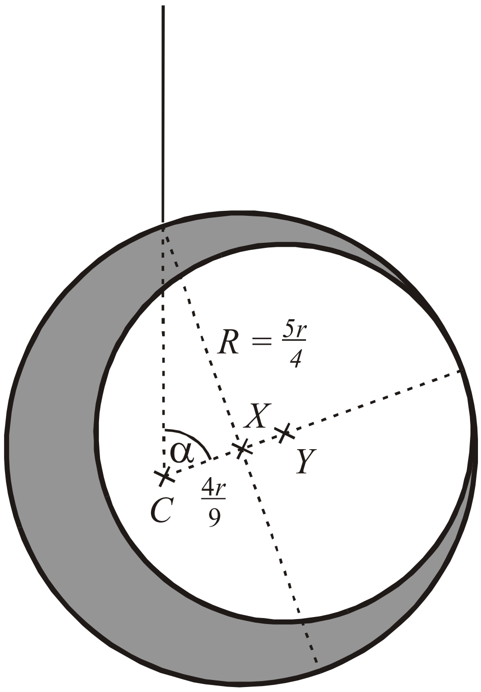
$$\left(R - r\right)\left(R + r\right)\frac{4}{9} - r\left(R - r\right) = 0$$

$$\left(R + r\right)\frac{4}{9} - r = 0$$

$$\frac{4}{9}R - \frac{5}{9}r = 0$$

$$R = \frac{5r}{4}$$

(ii) To solve this problem we must draw an appropriate diagram.



The diagram shows that

$$\tan\alpha = \frac{\left(\frac{5r}{4}\right)}{\left(\frac{4r}{9}\right)} = \frac{45}{16}$$

Hence $\alpha = 70.4°$


blacksacademy.net
is hiring Math tutors
blacksacademy.net/hiringtutors.php
ALL LEVELS CATERED FOR
Pay starting at £15 per hour
All preparation is done for you
Your English must be excellent
Math background required