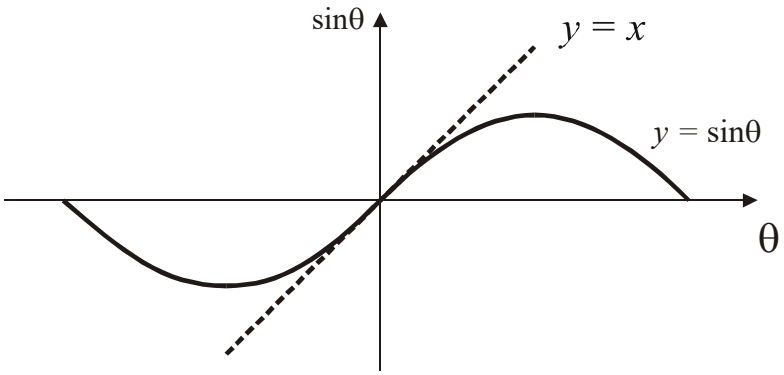
Small Angle Approximations

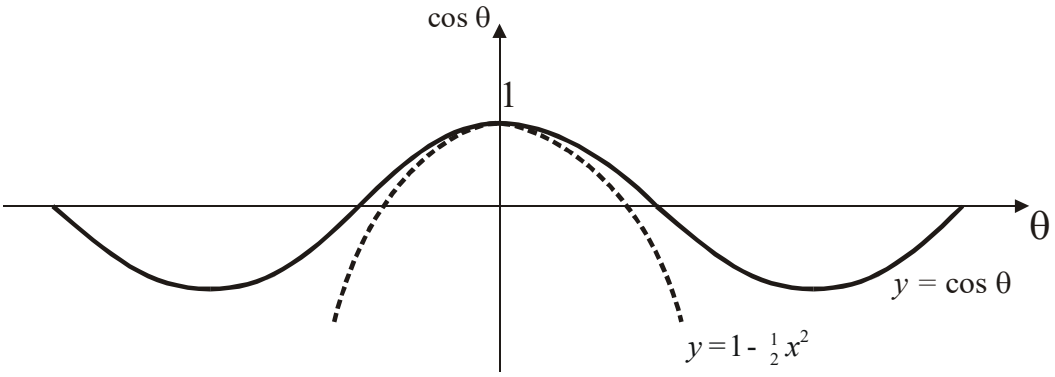
When angles are small, it is possible to approximate the trigonometric functions by other simpler functions.

The graphs of the trigonometric functions illustrate this.

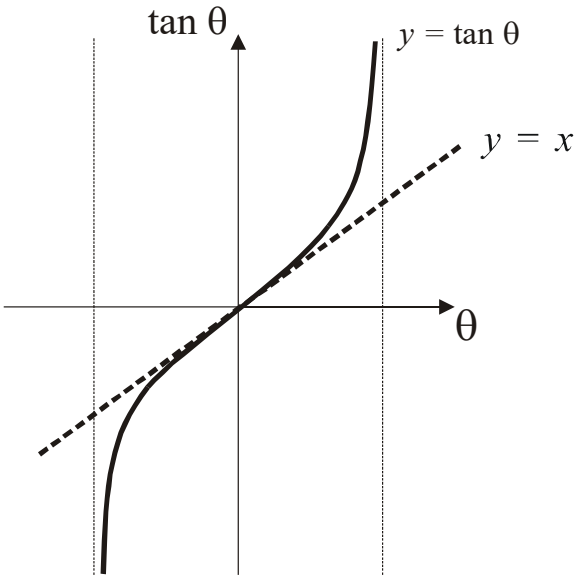
When θ is small, $\sin\theta \approx \theta$



When θ is small, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$



When θ is small, $\tan\theta \approx \theta$



These approximations can be used to find other approximations

Example

Given $\theta \approx 0$, find $\dfrac{1-\cos\theta}{\tan^2\theta}$.

Solution

$$\frac{1-\cos\theta}{\tan^2\theta} = \frac{1-\left(1-\frac{1}{2}\theta^2\right)}{\theta^2} = \frac{\frac{1}{2}\theta^2}{\theta^2} = \frac{1}{2}$$