Surds

What surds are

Surds are expressions involving the root symbol, $\sqrt{\ }$. For example $\sqrt{2}, \sqrt{3}, \sqrt{0.11}$ are surds. We are concerned here with processes that simplify expressions involving surds. The problem is that expressions involving surds can "clog up" the algebra, and techniques for simplifying surds are required to prevent this.

Factorisation

Numbers appearing under a root symbol should be simplified by factorisation wherever possible. For example

$\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2 \times \sqrt{3}$

Rationalising the denominator

Surds when they appear on the bottom of a fraction, (in the denominator), should be brought to the top of the fraction (the numerator) by multiplying the top and bottom of the fraction by that surd. This technique is known as *rationalising* the surd. For example

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Conjugates

Surds can appear in expressions added to integers and real numbers.

$2 + \sqrt{3}$ $\quad$ $3.28 + \sqrt{5}$ $\quad$ $\pi + \sqrt{7}$

The *conjugate* of such an expression is found by subtracting the surd from the real number part.

$2 - \sqrt{3}$ $\quad$ $3.28 - \sqrt{5}$ $\quad$ $\pi - \sqrt{7}$

When an expression containing surds is multiplied by its conjugate the surds disappear.

$\left(2 + \sqrt{3}\right) \times \left(2 - \sqrt{3}\right) = 4 - 2\sqrt{3} + 2\sqrt{3} - 3 = 1$

Conjugates are used to rationalise the denominator in cases when the denominator contains a surd added to a real number.

$$\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}$$

Example

(*a*) Simplify the following

$$\sqrt{32} + \sqrt{50} - \frac{6}{\sqrt{2}}$$

(*b*) Simplify $\frac{2-\sqrt{5}}{3+\sqrt{5}}$, expressing your answer in surd form.

Solution

(*a*)
$$\begin{aligned}\sqrt{32} + \sqrt{50} - \frac{6}{\sqrt{2}} &= 4\sqrt{2} + 5\sqrt{2} - \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= 4\sqrt{2} + 5\sqrt{2} - 3\sqrt{2} \\ &= 6\sqrt{2}\end{aligned}$$

(*b*) The question is asking you to rationalise the denominator

$$\begin{aligned}\frac{2-\sqrt{5}}{3+\sqrt{5}} &= \frac{2-\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} \\ &= \frac{6-2\sqrt{5}-3\sqrt{5}+5}{9-5} \\ &= \frac{11-5\sqrt{5}}{4} \\ &= \frac{11}{4} - \frac{5}{4}\sqrt{5}\end{aligned}$$

Summary

Simplifying surds

$$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$

Rationalising surds

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Technique for multiplying by conjugate

$$\frac{1}{2+\sqrt{3}} = \frac{1}{\left(2+\sqrt{3}\right)} \times \frac{\left(2-\sqrt{3}\right)}{\left(2-\sqrt{3}\right)} = 2-\sqrt{3}$$