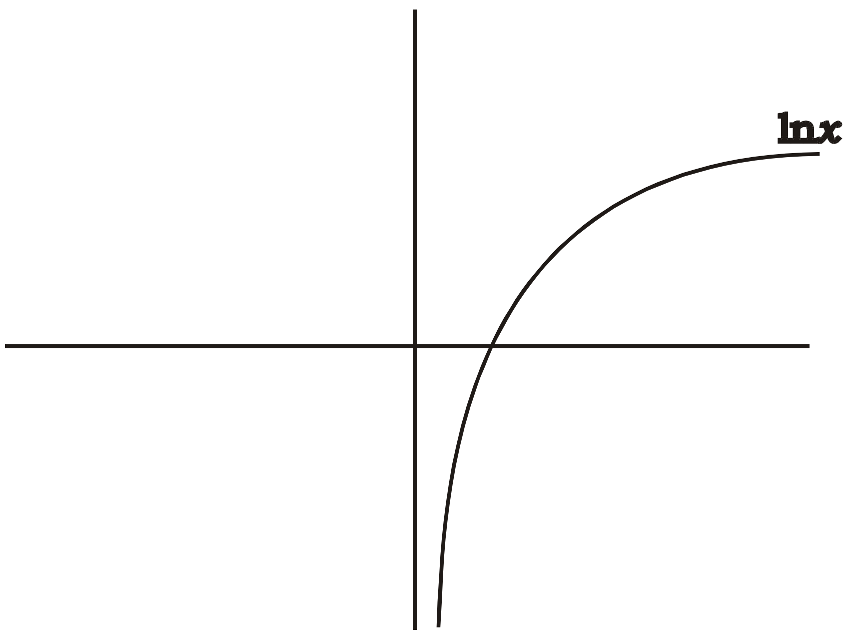
The Integral of $\frac{1}{x}$

The Integral of $\frac{1}{x}$ is given by:

$$\int \frac{1}{x}\,dx = \ln|x| + c$$

This expression involves the variable *x* enclosed within the modulus function. To explain why we introduce the modulus in this context we must first appreciate that $\ln x$ is undefined for $x \le 0$,



Hence, we cannot substitute negative values directly into $\ln x$. To avoid this we replace the negative expression $\frac{1}{x}$ in the integrand (that is, the expression to be integrated) by $-\frac{1}{-x}$. When this is integrated, we obtain $-(-\ln(-x))$ or $\ln(-x)$. This is equivalent to taking the natural logarithm of the modulus of *x* – that is, $\ln|x|$.

More explicitly

If $a,b<0$ then

$$\int_a^b \frac{1}{x}dx = -\int_a^b -\frac{1}{x}dx$$
$$= -\int_a^b \frac{1}{-x}dx$$
$$= -\left[-\ln(-x)dx\right]_a^b$$
$$= \left[\ln(-x)\right]_a^b$$

Since $a,b<0$ this is equivalent to $\ln|x|$.

If $a,b<0$ then

$$\int_a^b \frac{1}{x}dx = \left[\ln x\right]_a^b = \left[\ln|x|\right]_a^b$$

Either way

$$\int_a^b \frac{1}{x}dx = \left[\ln|x|\right]_a^b$$

Example

$$\int_{-2}^{-1} \frac{1}{x}dx = -\int_{-2}^{-1} -\frac{1}{x}dx$$
$$= -\left[-\ln(-x)\right]_{-2}^{-1}$$
$$= -\left(-\ln(1)+\ln(2)\right)$$
$$= -\ln(2)$$

The use of $|x|$ shortens this process

$$\int_{-2}^{-1} \frac{1}{x}dx = -\left[\ln|x|\right]_{-2}^{-1}$$
$$= \ln(1)-\ln(2) = -\ln(2)$$