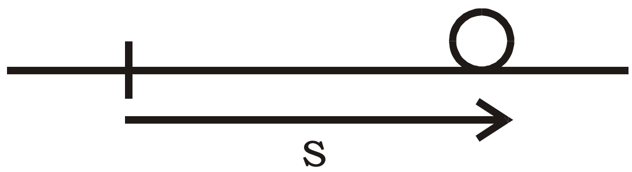
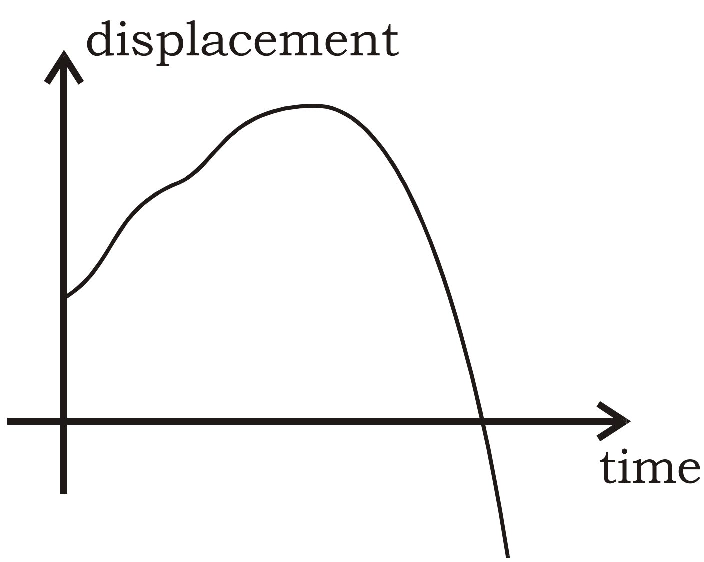
Velocity/Time and Displacement/Time Graphs

Displacement/Time Graphs

Here we study displacement in one direction only. That is, an object is allowed to move just backwards and forwards along a line. Displacement is measured as a distance, positive or negative, from an origin. The usual symbol for displacement is *s*.



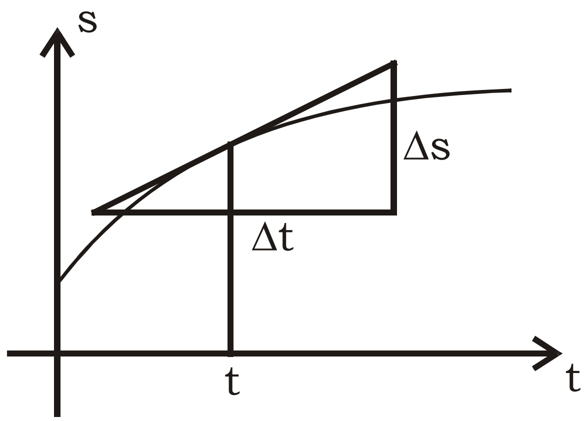
The displacement/time graph plots this displacement as a function of time.



Then

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time}}$$

The instantaneous velocity at time t_1 is the gradient of the displacement/time graph at t_1.



The symbol Δ stands for "change in". Hence, Δ*s* stands for "a change in *s*", or "a change in displacement".

The symbol δ stands for a "small change in", and the symbol *d* stands for an "instantaneous change in".

Thus:

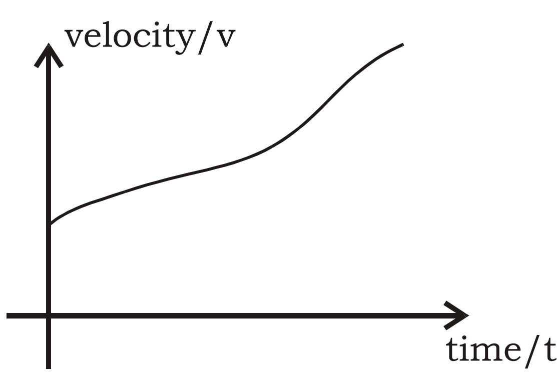
$$\text{instantaneous velocity at } t_1 = \frac{\text{change in displacement}}{\text{change in time}} = \frac{\Delta s}{\Delta t} \text{ of the gradient at } t_1$$

The instantaneous velocity at t_1 is the instantaneous rate of change of displacement with time. Hence, we can write:

$$\frac{ds}{dt} \text{ at } t_1 = \frac{\delta s}{\delta t} \text{ of the tangent at } t_1$$

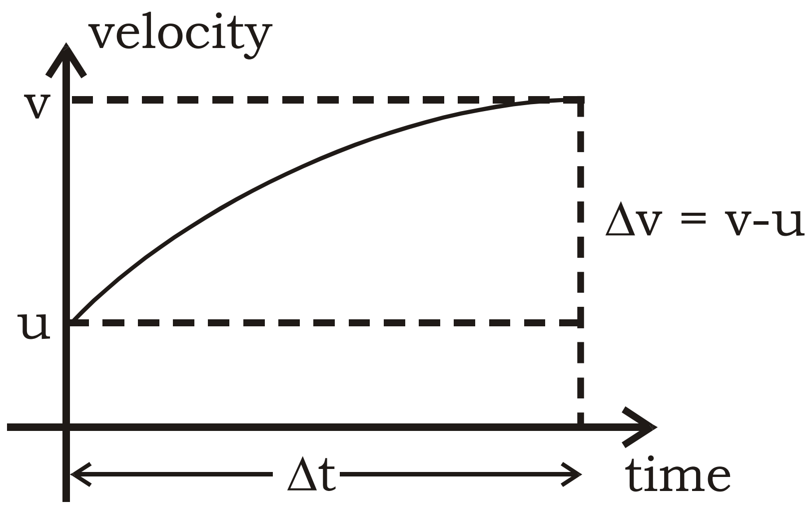
Velocity/Time Graphs

In addition to displacement/time graphs, we can also draw velocity/time graphs.



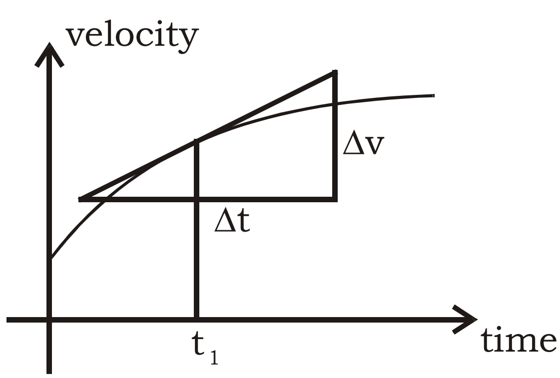
$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{change in time}} = \frac{\Delta v}{\Delta t}$$

Suppose a body has initial velocity u, and final velocity v, then:



$$\text{Average acceleration} = \frac{\Delta v}{\Delta t} = \frac{v-u}{t}$$

Note that strictly speaking we should always use symbols like Δt to stand for "change in time". However, we often drop the Δ part of this and use t alone.

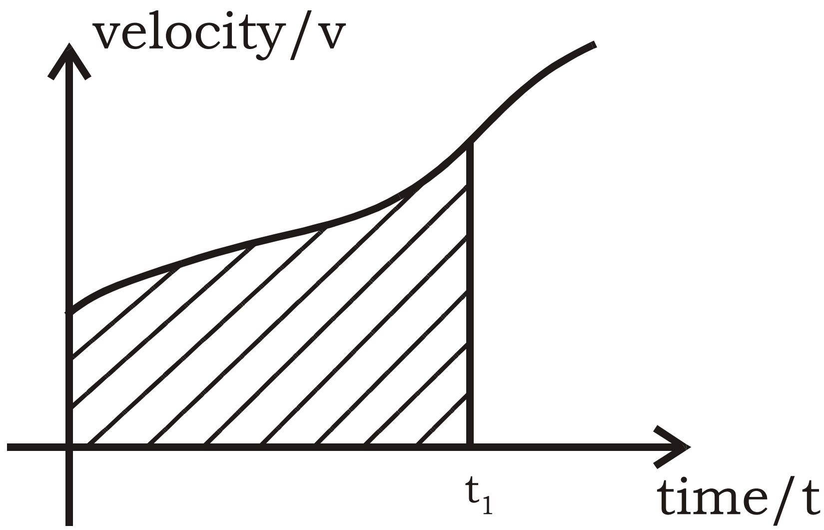


Instantaneous acceleration at t_1 = gradient of the velocity/time graph at t_1

$$a = \frac{dv}{dt} = \frac{\Delta v}{\Delta t} \text{ of the tanget to the v/t graph at } t_1$$

Area under a Velocity/Time Graph

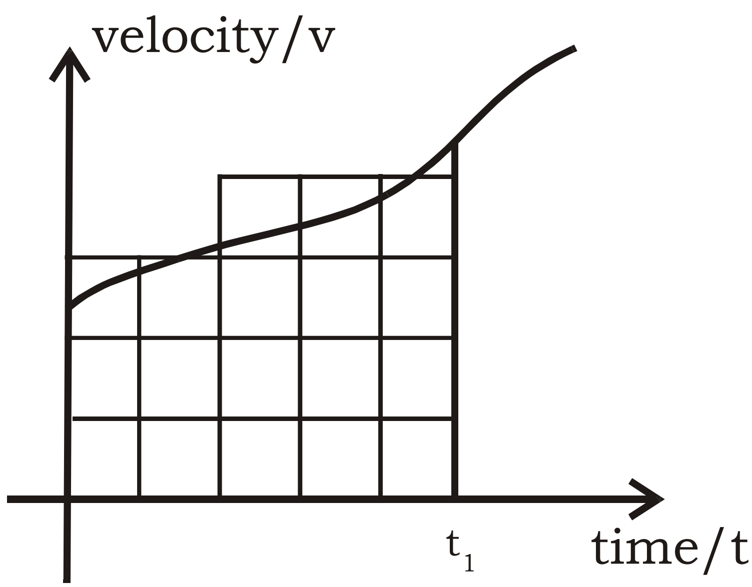
From the velocity/time graph we can also determine the displacement of the object.



Displacement to t_1 = area under the velocity/time graph to t_1

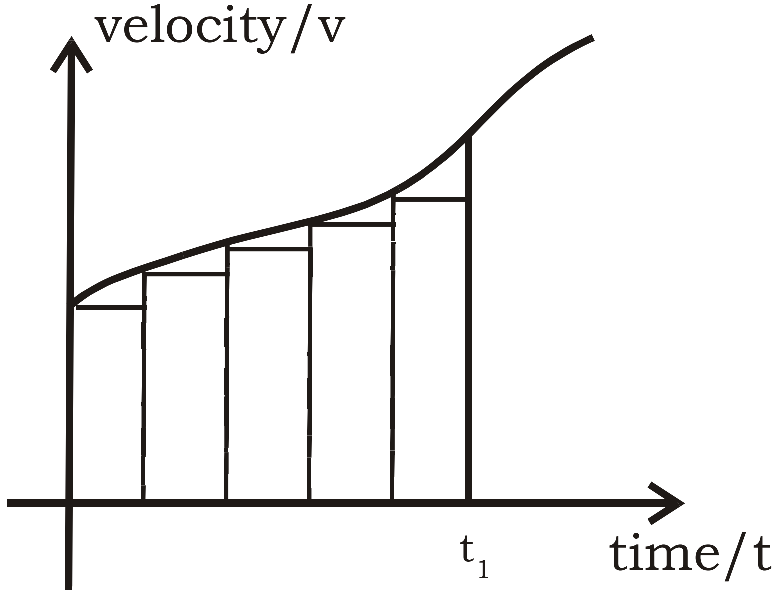
In some cases there are precise methods for calculating this area.

Alternatively, the area can be approximated from the graph by the technique of dividing the area into squares or rectangles and counting them



A square is counted in if more than half of it is in. The approximation gets better, the smaller the squares.

Here we illustrate the technique of dividing the area into rectangles:



The rectangles fit either above or below the curve, so one either over-estimates or under-estimates the area.