Weight and Free Fall

Prerequisites

You should be familiar already with the equations of uniform acceleration and with the concept of a force together Newton's first and second laws explaining the effect of a force on motion. Let us consolidate this.

Example (1)

A boy pulls a truck of mass 6 kg along a straight horizontal track with a force of 5 N. The friction opposing the motion is 2 N.

(*a*) What is the acceleration of the truck?

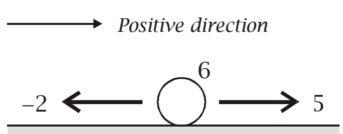
(*b*) If the boy pulls the truck from rest for 2 s

(i) How far does the truck travel in 2 s?

(ii) What is the final velocity of the truck after 2 s?

Solution

(*a*)



The resultant force is

$F = 5 - 2 = 3$ N

Newton's second law is

$F = ma$

Hence

$$a = \frac{F}{m} = \frac{3}{6} = 0.5 \text{ ms}^{-2}$$

(*b*) To solve these questions we need one of the equations of uniform acceleration.

(*i*) $s = ut + \frac{1}{2}at^2$

Here $u = 0$, $a = 0.5$ and $t = 2$

$s = \frac{1}{2} \times 0.5 \times 2^2 = 1\,\text{m}$

(*ii*) $v = u + at$

As before $u = 0$, $a = 0.5$ and $t = 2$

$v = 0.5 \times 2 = 1\ \text{ms}^{-1}$

You should be familiar with the concept of gravity

Example (2)

An apple is falling to the ground.

(*a*) Why is it falling and is its speed constant?

(*b*) Is the Earth falling towards the apple?

Solution

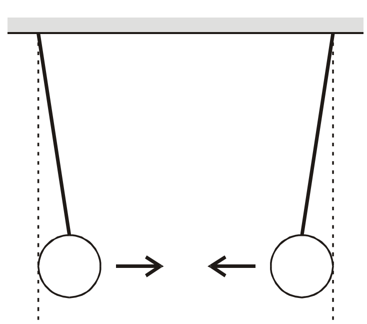
(*a*) The Earth's gravity exerts a force on the apple. The apple is attracted to the ground by virtue of this gravitational force. The force of the Earth's gravity is causing it to accelerate towards the ground; it is speeding up.

(*b*) Strange as it may sound, the Earth is also accelerating towards the apple. The apple also exerts a gravitational force of attraction on the Earth and causes the Earth to move towards the apple.

In order to understand the answer to part (*b*) of this question better, it is important to consider the question: exactly what is gravity?

The gravitational force

Gravity is a force produced by the attraction of two masses towards one another. It is a universal force that exists between all objects by virtue of their mass. For example, if two spheres are suspended next to one another by inextensible strings they will be attracted to one another, so they will not hang exactly vertically.



We do not notice this because the force of attraction is very small relative to the mass of an object. The spheres must be very massive for any deflection to be noticeable. However, very sensitive instruments can measure this deflection and so calculate the force of gravity. It is possible to determine the mass of a mountain by measuring its effect on a lead sphere suspended next to it

Thus the force of attraction between any two masses (1) depends on both masses, and (2) is equal and opposite.

Example (3)

Two masses A and B are in outer space where there is no friction. The mass of A is 1 kg and the mass of B is 100,000 kg. Ignore all other forces acting on A and B except the force of gravity between them.

(*a*) Is the magnitude of the force of gravity acting on A bigger, smaller or equal to the magnitude of the force of gravity acting on B?

(*b*) Find the ratio $\frac{p}{q}$ where p is the acceleration of A towards B and q is the acceleration of B towards A.

Solution

(*a*) The magnitude of the force of gravity acting on A is equal to the magnitude of the force of gravity acting on B. The two forces are equal, but act in opposite directions.

(*b*) Let F be the magnitude of the force acting on both masses. On substituting into Newton's second law $F = ma$ for both we get

$$100{,}000q = p$$

$$\frac{p}{q} = 100{,}000$$

The acceleration of A towards B is 100,000 times larger than that of B towards A.

A simplifying assumption

As the last example shows, because the magnitude of the force of gravity between two objects is equal this means that the effect (in the sense of bringing about acceleration) is greater on the smaller mass than on the larger mass. When one mass is much greater than the other, the acceleration of the larger mass can for practical purposes be ignored.

Example (4)

The mass of an apple is approximately 0.2 kg. The mass of the Earth is approximately 6×10^{24} kg. The acceleration of a falling apple is approximately 10 ms^{-2}. What is the acceleration of the Earth towards a falling apple?

Solution

The force is the same (equal but opposite) in both cases so on substituting into $F = ma$ we have

mass of Earth $\times$ acceleration of Earth = mass of apple $\times$ acceleration of apple

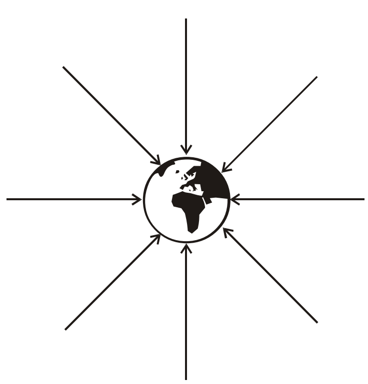
$$6\times10^{24}\times a = 0.2\times10$$

$$a = \frac{0.2\times10}{6\times10^{24}} \approx 3\times10^{-25}$$

So the acceleration of the Earth is minute compared to the acceleration of the apple. It is so minute that *for practical purposes* we can ignore it. This is a simplifying assumption. It simplifies things because in practical questions instead of considering the motion of two objects (here the apple *and* the Earth) we only have to consider the motion of one object (just the apple).

The gravitational field and a second simplifying assumption

A mass exerts a *gravitational field*. This is a region surrounding a mass where the effect of its gravitational force is significant. The situation we are imagining is something like that of the Earth and the apple, where the Earth is massive in comparison to the apple. So the apple exists in the gravitational field of the Earth, and we imagine the Earth to be stationary and only the apple (when it is falling) to be moving. The apple falls towards the centre of the Earth – or we should say the *centre of gravity* of the Earth to be precise. The apple falls along straight lines towards this centre of gravity. This means that we can sketch the gravitational field of the Earth (or any mass) as follows.



The strength of the gravitational force decreases as the distance from the centre of gravity increases. In the above diagram this means that the force of gravity acting on a 1 kg mass on the surface of the Earth is much greater than the force of gravity acting on the same mass but located far away in outer space. However, this is all a question of relative distances. The Moon is about 400,000 km away from the surface of the Earth; the radius of the Earth is about 6,000 km, so over these distances the strength of the gravitational force varies considerably. In comparison even if you were to go to the highest point on the surface of Earth, Mont Everest at approximately 8 km above sea level, the gravitational force would be hardly different from that at sea level. So we introduce a second simplification, and pretend that the gravitational force at the surface of the Earth remains constant.

Weight and the gravitational field constant

Let us assume that we are dealing only with the gravitational force acting on a small mass situated in the gravitational field of a much larger mass, such as the Earth or Moon. The field produced by the large mass may be regarded as constant, but the size of the force acting on the smaller mass will vary with the size of the smaller mass. It is a law that this force is proportional to the size of the smaller mass.

Example (5)
A mass of 2 kg, situated in the gravitational field of a planet of large mass, experiences a force of 10 N. What would be the gravitational force acting on a mass of 4 kg situated at exactly the same position in this gravitational field?

Solution
The force is proportional to the mass. The 4 kg object is twice the mass of the 2 kg object, so the force acting on the 4 kg object is twice that acting on the 2 kg object. The answer is 20 N.

Example (5) continued
Assuming that the 2 kg and 4 kg masses are free to move, what is the acceleration of each mass that is produced by the gravitational field of the planet? In what direction would each mass move?

Solution
From Newton's second law we have

$a = \frac{F}{m}$

For the first mass this gives

$a = \frac{10}{2} = 5\,\mathrm{ms}^{-1}$

For the second mass this gives

$a = \frac{20}{4} = 5\,\mathrm{ms}^{-1}$

It may be noted immediately that the answer is the same in both cases. Both objects would move in the same direction – that is, along a straight line directly towards the planet's centre of gravity.

What this example demonstrates is that the acceleration produced by a gravitational field acting on any smaller mass at a given distance from the centre of the field (the centre of gravity) is the *same* regardless of the mass of the smaller object. This acceleration is called the *acceleration due to gravity* and has a *constant* value for a given gravitational field at a given distance from the gravitational field's centre. It is a property of the gravitational field and is therefore also called the *gravitational field constant*, so it has two names and both names are interchangeable. This *gravitational field constant* (or *acceleration due to gravity*) is represented by the symbol g.

We also have a special name for the force acting on an object located in a gravitational field. This is *weight*. Often we are dealing only with the weight of bodies on or near the surface of the Earth or some other planet of planetary body, such as the Moon. Weight is a force whereas mass represents the amount of substance of an object. The weight of an object is measured in Newtons and it varies from one gravitational field to another, or changes if the distance of the object from the centre of gravity changes significantly. Mass remains the same regardless of where the object is located. We use the symbol W for weight.

Because weight is a force it causes objects to accelerate – and as already stated, this acceleration is called *acceleration due to gravity.* So we may substitute weight = W and acceleration = g (acceleration due to gravity) into Newton's second law, $F = ma$, to get

$W = mg$

In words this is

$\text{weight} = \text{mass} \times \text{gravitational field constant}$

or alternatively

$\text{weight} = \text{mass} \times \text{acceleration due to gravity}$.

The gravitational field constant may be given with *two* units. As the field constant it is quoted in Newtons per kilogram, Nkg^{-1}; as acceleration due to gravity it is quoted as metres per second squared, ms^{-2}.

Example (6)

The gravitational field constant of the Earth is approximately 9.8 Nkg^{-1}. The gravitational field constant of the Moon is approximately 1.6 Nkg^{-1}. What is the weight of a 3.5 kg object (*a*) on the surface of the Earth, and (*b*) on the surface of the Moon?

Solution

(*a*) At the surface of the Earth

$W = mg = 3.5 \times 9.8 = 34.3\,\text{N}$

(*b*) At the surface of the Moon

$W = mg = 3.5 \times 1.6 = 5.6\,\text{N}$

As this example illustrates objects have considerably less weight on the surface of the Moon than on the surface of the Earth. They are not so heavy there.

The value of *g*

The value of the gravitational field strength for the surface of the Earth is determined by experiment to a certain level of accuracy. In most questions at this level you may take it to be $g = 9.8\ \text{Nkg}^{-1}$. The approximation $g = 10\ \text{Nkg}^{-1}$ is also used. To a higher level of accuracy $g = 9.81\ \text{Nkg}^{-1}$. The standard measurement to 5 decimal places is $g = 9.80665\ \text{Nkg}^{-1}$. However, this is an average value, for the strength of the Earth's gravitational field does vary with the thickness of the Earth's crust and its density, so there are places on the surface of the Earth where things are lighter than others. We assume in our questions that $g = 9.8\ \text{Nkg}^{-1}$ and is constant.

Free fall

When an object is falling in a gravitational field this is called *free fall*. It falls under its own weight and this causes it to accelerate. If it is falling in air it is subject to other forces, such as air resistance, but we also simplify problems by ignoring air resistance. Then an object in free fall is subject only to gravity and its weight is given by

$W = mg$

Example (7)

A sphere of mass m is gently pushed off a ledge. It falls for a total of 2 s when it strikes the ground. Ignoring air resistance find its velocity at the moment of impact.

Solution

In this question it is not necessary to know the mass of the object because it is in free fall. Its acceleration is $g = 9.8 \text{ ms}^{-2}$. Let the change in velocity be Δv and the change in time $\Delta t = 2 \text{ s}$. From the definition of acceleration as change in velocity per unit time

$$a = \frac{\Delta v}{\Delta t}$$

we get

$$\begin{aligned} \Delta v &= g \times \Delta t \\ &= 9.8 \times 2 \\ &= 19.6 \end{aligned}$$

The object started at rest, so this *increase* in velocity is the same as its velocity at impact.

$\text{velocity at impact} = 19.6 \text{ ms}^{-1}$.

We remind you that displacement, velocity and acceleration are vectors, with both magnitude and direction. In certain questions it is important to consider the direction of motion.

Example (8)

A missile is projected upwards from the edge of a cliff with an initial velocity $u = 24 \text{ ms}^{-1}$. Assuming that the missile is free to fall over the edge of the cliff and does not reach the ground below, find its velocity 6 s later. Ignore air resistance.

Solution

Here the initial velocity of the stone means that the stone is travelling upwards at the start. However, its weight acts downwards. So we have to introduce negative sign (–) somewhere to mark this distinction. Let the positive direction be upwards, then the acceleration is downwards, given by $g = -9.8 \text{ ms}^{-2}$. The change in velocity over 6 s is

$$\begin{aligned} \Delta v &= g \times \Delta t \\ &= -9.8 \times 6 \\ &= -58.8 \text{ ms}^{-1} \end{aligned}$$

So

$\text{final velocity} = \text{initial velocity} + \text{change in velocity}$

$$\begin{aligned} v &= u + \Delta v \\ &= 20 - 58.8 \\ &= -38.8 \text{ ms}^{-1} \end{aligned}$$

The negative sign means that the missile is travelling downwards 6 s after projection.

Questions may require you to apply your prior knowledge of the equations of uniform acceleration.

Example (9)

A small ball is fired vertically upwards with a speed of 12.6 ms^{-1}.

(*a*) Find the greatest height of the ball above the point of projection.

(*b*) Calculate the time taken for the ball to return to the ground.

(*c*) Determine the speed and direction of the motion of the ball 1.5 s after projection.

Solution

Once the ball has been fired it is moving under gravity. The question does not mention air resistance, so we ignore it. Defining the positive direction to be upwards the ball's initial velocity is $u = +12.6 \text{ ms}^{-1}$ and its acceleration throughout is -9.8 ms^{-2}.

(*a*) At the point of greatest height the particle is at a moment of instantaneous rest where $v = 0$. The appropriate equation is $v^2 = u^2 + 2as$ and on substitution we get

$$0 = (12.6)^2 - 2 \times 9.8 \times s$$
$$s = 8.1 \text{ m}$$

(*b*) The appropriate equation is $s = ut + \frac{1}{2}at^2$ with $s = 0$.

$$0 = 12.6t + \frac{1}{2} \times 9.8t^2$$
$$t(12.6 - 4.9t) = 0$$
$$t = 0 \text{ or } t = \frac{12.6}{4.9} = 2.5714... = 2.6 \text{ s } (2.\text{s.f.})$$

(*c*) The appropriate equation is $v = u + at$ with $t = 1.5$.

$$v = 12.6 - 9.8 \times 1.5$$
$$= -2.1 \text{ ms}^{-1}$$

The speed is 2.1 ms^{-1} and the direction is downwards.

Questions may require you to work backwards from information about the motion of an object to deduce something about the forces that have brought this about.

Example (9)

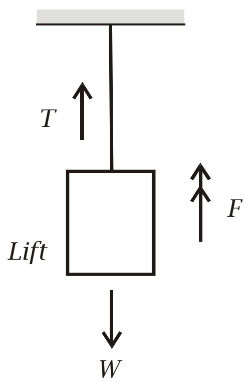
A lift of mass 825 kg accelerates upwards at a speed of 2 ms^{-1}. Find the tension in the cable.

Solution

The acceleration is a = 2 ms^{-1}. Therefore, the force required to produce this is given by Newton's second law

$$\begin{aligned} F &= ma \\ &= 825 \times 2 \\ &= 1650 \text{ N} \end{aligned}$$

But this is *not* the tension in the cable, as the cable must not only accelerate the lift but also support its weight. The force F is the resultant arrived at from subtracting the weight W of the lift from the tension T.



$$F = T - W$$
$$T = F + W$$

The weight is $W = mg = 825 \times 9.8 = 8085$ N

Hence $T = 1650 + 8085 = 9735$ N